REAL OPTIONS IN THE ENERGY MARKETS

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Chapter 1

Introduction

1.1 Background

About four decades ago, professor Stewart Myers coined the term "real options" in a well-known paper [Mye77], observing that corporate investment opportunities can be viewed as call options on real assets. Since then, numerous researchers addressed investment opportunity under uncertainty by using a real options approach. This research has established a theoretical framework for the modeling and pricing of real options (see for example [DP94], [Tri96], [AK98], etc.), and investigated a variety of real options. The real options' applications have been extended from natural resources investment [BS85] to a wide range of investment problems [CA03]. The business community also shows a growing interest in real options. Many world famous companies, e.g., Boeing [MN04], BP [LM97], etc., have adopted the real options technique for project valuation and investment decision-making.

Today, at least in academia, real options theory has been widely accepted as an innovative tool for capital planning and asset valuation. The real options theory argues that the asset operator can avoid downside scenarios of an investment and maintain the upside profit by responding appropriately to the outcome of the invested project. This is a logical and intuitive argument, since any rational agent will perform this way if he or she is capable to do so. This is also where the real options theory deviates from the traditional discounted cash flow (DCF) method. The DCF method assumes that the company has to accept all the possible outcomes of a project as a whole once the investment has been decided. Furthermore, the DCF method views any investment as a "now-or-never" opportunity, while in real options theory, the investor may wait for some time until additional favorable information validates the investment commitment [MS86].

Real options are embedded in many assets and projects, although the value of these options is not always recognized. The risk of underestimating the asset value always exists.

1.2 Motivation

Early real options applications were mostly found in the energy industry. And since then, the energy industry has been a fertile field where an enormous literature has been spawned. The energy industry has several reasons to stimulate real options applications.

Firstly, the energy industry is characterized by its intensive capital expenditure. Examples are oil field development and power plant investments. Huge investment expenditure calls for reliable valuation and decision-making tools.

Secondly, many energy assets bear certain kinds of operational flexibilities [Ron02]. These operational flexibilities, along with the investment opportunities, are the sources of option value that are embedded in energy assets. More examples will be offered in Chapter 2 and Chapter 3.

Thirdly, the outputs of the industry are mostly traded commodities. The petroleum products trading has existed for decades. Gas and electricity have been recently deregulated, and they are now traded in exchanges in US, Canada, Europe and Australia. Some energy-related derivatives, such as weather derivatives, emission rights, come into trading as well. The existence of rich market data is important to make reasonable assumptions when modeling the uncertainties. It will be difficult to assess the value distribution that may result from a non-commodity related project.

Finally, companies in energy industries have the engineering culture to adapt mathematical models.

This thesis focuses on the real options applications in the thermal power sector. However, in the general introduction to real options applications to energy industry, cases in oil and gas sectors are also presented. In the domain of thermal power management, we found the following research opportunities.

Firstly, while various stochastic models are available to model the electricity spot prices, it is valuable to choose a best-performing model for a specific market via empirical tests.

Secondly, while many researchers have modeled a thermal power plant as a string of options, we need to develop more realistic models which can incorporate the operational characteristics of a power plant. A load-servicing power plant faces not only market risk, but also volumetric risks[1], which include both the supply side and the demand side risks. How the volumetric risk factors can affect the power plant value is still poorly understood.

Thirdly, the spark spread[2] is the most important driver of an investment in a power plant. Although the spark spread is not directly observable in the markets, we can construct a spark spread time series from the observed electricity and gas prices. Based on the spark spread process, we can value a power plant and various investment opportunities.

This thesis aims to contribute to the real options literature by addressing the above-mentioned aspects.

1.3 Structure

The remainder of the thesis is organized as follows.

Chapter 2 presents an overview of the real options theory. Classical types of real options and the mathematical approaches to pricing real options are discussed. Chapter 3 reviews real options modeling in the energy industry, highlighting the cases in the oil and gas asset investment, power plant valuation, and gas storage valuation. In Chapter 4, we study the empirical properties of electricity prices in a deregulated market and survey the mathematical tools used for spot price modeling. Empirical tests with the Dutch and German market data are performed with two spot price models. Chapter 5 analyzes the option-based valuation of a gas-fired power plant and how the value is affected by the operational constraints and customer load uncertainty. In Chapter 6, we value a power plant investment licence as an investment option. Modeling the spark spread with a one-factor model, we derive the valuation formula for both base- and peak-load power plants and price various investment options. Finally, Chapter 7 summarizes the main conclusions, discusses relevant issues and suggests research directions in the future.

[1] In Chapter 5, we will give a definition of volumetric risks in a power plant.

[2] In Chapter 2, we will give a definition of spark spread and spark spread option.

From Chapter 2 to Chapter 6, each chapter itself is actually an independent working paper addressing one specific issue encompassing real options in the energy markets. Chapter 1 and Chapter 7 are a general introduction and a summary of the whole thesis, respectively.

Chapter 2

Real Options Theory

2.1 NPV and DCF

The traditional method in capital budgeting is based on the discounted cash flow (DCF) method. The value of a project is measured by the net present value (NPV), i.e., the present value of the future cash flows minus the initial investment outlay. The optimal investment rule is to invest if and only if the NPV of the project is positive.

The DCF method has substantial drawbacks. The first drawback is that it is not easy to estimate future cash flows of a project, because the price of the output, the production rate, and the investment cost, are usually not deterministic. Secondly, the DCF method uses a discount rate to reflect the risks of these cash flows and this discount rate is unavoidably subject to estimation errors. Thirdly, the DCF method does not take into account the managerial flexibilities which are embedded in a project, such as the option to delay an investment waiting for some uncertainties to disappear, the option to expand or contract the scale of a project, the option to change the input or output of the project, the option to abandon the project if the outcome is extremely negative. In a DCF valuation, implicit assumptions are made on both the expected scenario of the operating cash flows and the managers' passive commitment to a certain operating strategy [Tri96].

2.2 Real Options Theory

Real Options Theory (ROT) or Real Options Valuation (ROV) approach, is based on the analogy between investment opportunities and financial options. A real option is a right, but not an obligation, to do something for a certain cost within or at a specific time. With the ROV method, a project is considered an option on the underlying cash flows and the optimal investment strategies are just the optimal exercise rules of the option.

The ROV approach overcomes the shortfalls of the DCF method discussed in Section 2.1. The ROV considers all possible price paths for the underlying project value or underlying commodity prices and assumes a distribution for the underlying prices rather than a deterministic price assumption. Moreover, given the underlying asset being tradable and replicable, there is no need to estimate the risk-adjusted discount rate since options are valued using the risk-free interest rate under the risk-neutral measure. Finally, the ROV allows for the consideration of possible options that are embedded in investment projects, in which the managers have the flexibility to respond to the outcome of uncertainties.

The managers' abilities to react to market conditions tend to expand the value of the investment project by maintaining or improving the upside potential and limiting the downside loss. As a result, the real options method may accept a project with a negative NPV. With an option to wait, the real options method may delay the execution of the investment activity.

The ROV is not considered an alternative valuation method to the DCF, but rather as an expanded DCF [Tri96]. In the expanded DCF, the value of any investment consists of two components: the traditional (static or passive) NPV of the direct cash flows, and the option value of the managerial flexibility. The difference between the traditional DCF value and the real options value is the value of the options that are embedded in the investment project.

2.3 Real Options versus Financial Options

In this section, we define financial options and introduce the three methods for financial pricing. Next, we give a definition of real options and study the analogies between financial options and real options.

2.3.1 Financial Options and Pricing

In the financial markets, an option provides the holder with the right to buy or sell a specified quantity of an asset at a fixed price, which is called a strike price or an exercise price, at or before the expiration date of the option. Since it is a right and not an obligation, the holder can choose not to exercise the right and allow the option to expire.

A financial option is characterized by the right, but not the obligation, to perform a transaction. The underlying assets for financial options may be stocks, stock indices, foreign currencies, debt instruments, commodities, and futures contracts. The two basic types of options are call and put. A call option gives the holder the right to buy an underlying asset for a specified exercise price within or at a specified time. A put option gives the holder the right to sell the underlying for a specified exercise price within or at a specified time. The specified time, i.e., the life time of the right, is also called the maturity time. Financial options are also categorized by the time when they can be exercised. An American option can be exercised at any time up to the expiration date. A European option can be exercised only on the expiration date.

The key property of an option is the asymmetry of the payoff. An option holder can take advantage of the upside risks and limit the loss to the price of the option. In Figure 2.1, the payoffs of a call and a put option on a stock price are given as an example.

If the stock price at maturity is lower than the strike price, the holder of a call option will not exercise it, the loss is then limited to the price to purchase the option. This is shown in the left panel of Figure 2.1. If the stock price at maturity is higher than the strike price, the holder would exercise the option, and obtain a payoff equal to the realized stock price minus the strike price. There is no upper bound of the payoff but the lower bound of the payoff is zero. The maximum loss is equivalent to the original purchase price of the option.

Options are often categorized by their moneyness. If an immediate exercise of an option leads to a profit, we call the option "in-the-money"[1]. To the opposite, if an immediate exercise of the option leads to a loss, the option is "out-of-the-money". An option whose underlying spot price equals to the strike price is thus "at-the-money".

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[1] Before maturity, a European call option cannot be exercised. We still call it "in-the-money" if the stock price is higher the strike price.

The closed-form valuation of an option was first found by Black and Scholes [BS73], and Merton [Mer73]. Their option pricing models are based on some assumptions, the most important one is that the underlying price follows a Geometric Brownian motion, which implies that the stock price follows a lognormal distribution. Denote S as the underlying stock price, K as the strike price, T as the time to maturity, σ as the volatility of the stock price, and r as the risk-free interest rate. The Geometric Brownian motion assumption implies

$$dS = \mu S dt + \sigma S dz \tag{2.1}$$

where μ is the growth rate of the price, and $\{z_t, t \geqslant 0\}$ is a Brownian motion process.

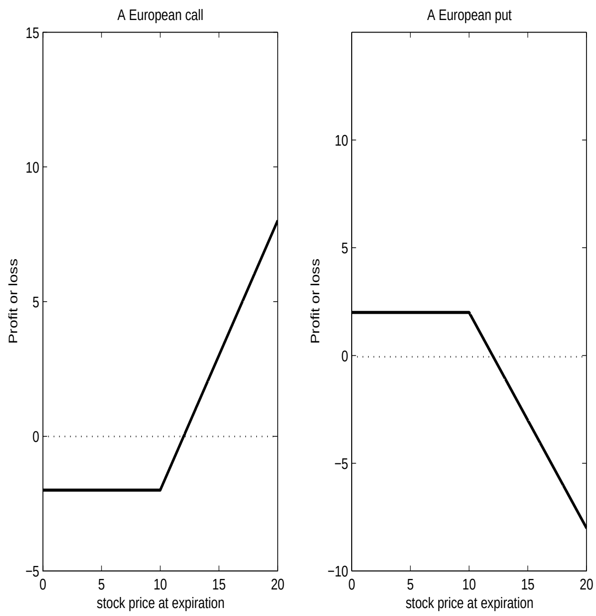

Figure 2.1: Payoff of European call and put options

Then the value of a European stock call option at time t is given by the Black-Scholes formula

$$C = SN(d_1) - Ke^{-r(T-t)}N(d_2) \tag{2.2}$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}} \quad (2.3)$$

$$d_2 = d_1 - \sigma\sqrt{T - t} \quad (2.4)$$

and $N(.)$ is the cumulative probability of a standard normal distribution function[2].

Standard European or American options are sometimes called Vanilla options. By contrast, exotic options are derivatives with more complicated payoffs. Exotic options valuation techniques are of special interests to real options researchers because real options usually possess more complicated payoff structures than standard European or American options.

There are various exotic options. To exemplify, we have: Compound options, which are options on options; Barrier options, which are options where the payoff depends on whether the underlying asset's price reaches a certain level within a certain period of time; Asian options, which are options where the payoff depends on the average price of the underlying asset during part or all life of the option; Bermudan options, which are very close to American options but the early exercise are restricted to certain dates within the life of the option, and Basket options, which depend on the underlying of a portfolio of assets.

If the payoff of a derivative depends not only on the final value of the underlying asset, but also on the path followed by the price of the underlying asset, we call this derivative path dependent. For example, Asian options and barrier options are both path-dependent options. Only in rare cases do we have analytic solutions for path-dependent options. Examples are found for Asian options [KV90] and Barrier options [Bjo04]. In many cases, we can use binomial trees to cope with the path dependency. Finally, Monte Carlo simulation is the approach that can always be tried to value path-dependent options when analytic results are not available.

2.3.2 Analogy to Financial Options

A real option is the right, but not the obligation, to take an action (e.g., deferring, expanding, contracting or abandoning) at a predetermined cost called the exercise price, for a predetermined period of time - the life of the option.

[2] A standard normal distribution has a mean of 0 and a standard deviation of 1.

A real option is a certain kind of flexibility that is embedded in a real asset or investment project. A real option resembles a financial option in many ways. Hence, there exists a close analogy between financial and real options.

As an example, the investment opportunity in a project can often be seen as a call option on the present value of the expected cash flows from the investment. The analogy is shown in Table 2.1.

Call option on stock	Real option on a project
Current stock price	Gross present value of expected cash flow
Exercise price	Investment cost
Time to maturity	Time until the opportunity expires
Stock price volatility	Project value uncertainty
Risk-free interest rate	Risk-free interest rate
Dividend	Cash flow or value leakage

Table 2.1: Analogy between financial options and real options

Similarly, an option to abandon a project is analogous to a put option on the project's value. The exercise price is the salvage value of the equipment. The other analogies are the same as in Table 2.1.

However, real options are more complex than financial options. The major difference is that the underlying assets of real options are not tradable. Non-tradable real assets may earn a return below the equilibrium rate of return expected in the financial market. The rate of return shortfall necessitates a dividend-like adjustment. In option pricing, we mostly apply a risk-neutral valuation, by using the certainty-equivalent or risk-adjusted growth rate, which is equal to the actual growth rate minus an appropriate risk premium. More details about risk-neutral valuation of real options will be addressed in Section 2.5.

2.4 Taxonomy of Real Options

By the type of flexibility that the operators may have in the operation of an asset or a project, real options are classified into several categories. Following the rich literature, we briefly introduce: the option to wait, the option to abandon, the option to expand/extract, the option to switch, compound options, and rainbow options. In each option type, an example in the energy applications is given. More details in the classification of real options are found in [Tri96], [AK98] and [CA03].

The option to wait (or a deferral option) is found in most investment projects. It is an American call option on the project value at the exercise price- the money invested in getting the project started. The lease contract on undeveloped oil field is a typical example where the option to wait is remarkably valuable. The unfolding of part of uncertainties in the project is thought as the source of the option value.

The option to expand/contract (scale option) is an American type option. With this kind of flexibility, the operating scale can be altered in response to the market conditions. The option to expand the scale of a project by committing more investment is an American call. The option to scale back by selling a fraction of a project is an American put. In extreme cases, the production may be halted and restarted. Moreover, the owner of such a flexible asset may have more than one specified time points to exercise the option. In this case, we call it a Bermudan option.

The option to abandon a project for a fixed price is formally an American put. If market conditions decline severely, the owner of the project can give up the project permanently and realize the salvage value. This option may be valuable for some capital intensive investments.

The option to switch may contain a wide variety of flexibilities, which allow the owner of an asset to switch at a cost between two modes of operation. According to price or demand changes, the owner can change the output mix of a facility. For example, a CHP[3] plant can decide the weight of power and heat in its production. Alternatively, the same output can be produced by using different types of inputs. For example, an IGCC[4] power plant can choose to burn the cheapest fuel. Switch options can be viewed as portfolios of American call and put options.

Most real-life projects fit into the category of *compound options*, which is also known as options on options. That is because a collection of various options are often involved in real-life investments. In phased investments, each phase is an option that is contingent on the earlier exercise of other

[3] A combined heat and power (CHP) plant is flexible in its output mix, which consists of heat and electricity.

[4] An Integrated Gasification Combined Cycle (IGCC) power plant is flexible in the fuels to use; apart from the synthesis gas, it may fire oil coke, heavy refinery liquid fuels, natural gas, biomasss, and urban solid waste, among others.

options. A good example is given by upstream petroleum investment, where the option to produce oil depends on the option to develop the reserve. In sequence, the development option depends on the decision made in the exploration stage.

Simple options have only one source of uncertainty, namely, the price of the underlying asset. However, most real options' value is driven by multiple sources of uncertainty. These options are called *rainbow options*. As an example, an undeveloped oil reserve can be considered a rainbow option, in that the owner's choice whether to develop the reserve is affected by two sources of uncertainty. The first is obviously the price of oil, and the second is the quantity of oil that is in the reserve. To value this undeveloped reserve, we can make the simplifying assumption that we know the quantity of the reserves with certainty. Also, uncertainty about the quantity can be accounted for as a second factor in more complex models. The spark spread option is another example, where the electricity and gas prices are the two uncertainties.

The classification of real options provides an easy intuition for understanding the flexibility features in an asset. However, this taxonomy is not rigid. A real-life real option does not necessarily fall into only one of the listed categories. An example is a peak power plant, which we discussed above is regarded there as the option to alter its operational scale. Note that it can also be easily regarded as an option to switch between two operational modes. Fortunately, this ambiguity in real options taxonomy does not affect our quantitative analysis of the option value, since when we price these options, it is the generated cash flows in different scenarios that matters, not the name we call them.

2.5 Approaches to Real Options Pricing

The valuation of a real option can be viewed as an investment optimization problem under uncertainty. The idea is to maximize the NPV of the asset incorporating the relevant managerial flexibility, but subject to operational constraints.

2.5.1 Analytical Approximation

Two parallel methods, namely dynamic programming and contingent claims, can be exploited to address a real options optimization problem.

Dynamic Programming

Dynamic programming is a standard technique for dynamic optimization. At each point of time, we split the whole sequence of decisions into two parts: the immediate decision, and the remaining decisions, all of whose effects are encapsulated in the continuation value. To find the optimal sequence of decisions we work backwards from the last decision point. At the last decision point, there is no continuation value, so the best choice is easy to make. Then at the point before that one, we know the expected continuation value and therefore can optimize the current choice.

We use the *state variable* to describe the market and/or physical conditions of the asset. Let x_t be the state variable at any date or discrete time period t, and x_t is an Markovian process. At each period t, the asset owner is able to make some choices for the operation of the asset. We represent these choices with the *control variable* u_t. We denote the immediate profit flow as $\pi_t(x_t, u_t)$ and $F_t(x_t)$ the value of the asset. Further assuming a constant discount rate ρ, then at each time point t the asset value must satisfy the following Bellman equation[5]:

$$F_t(x_t) = \max_{u_t}\{\pi_t(x_t, u_t) + \frac{1}{1+\rho}E_t[F_{t+1}(x_{t+1})]\} \tag{2.5}$$

where $E_t[.]$ is the expectation operator at time t according to a real world measure.

Implicit in this equation are the assumptions that the remaining choices $u_{t+1}, u_{t+2}, ...,$ are already optimal in the continuation value. So only the immediate choice u_t is left for the optimal decision. The first term on the right-hand side is the immediate profit, and the second term is the continuation value. The objective of the problem is to decide for the period t the optimal control variable u_t, which maximizes the sum of these two components.

In continuous time, a deferral investment option can be approximated by a binary decision problem, i.e., the firm is left to choose to invest or not at

[5] A Bellman equation is also called an optimality equation or a dynamic programming equation, named after its developer Richard Bellman. An introduction to Bellman equation and dynamic programming can be found in [BD62].

each time t. Assume the state variable follows a continuous GBM as defined in equation (2.1), i.e.,

$$\frac{dx_t}{x_t} = \alpha dt + \sigma dz_t \tag{2.6}$$

The firm can either receive a cash flow $\pi(x,t)$ by continuing with waiting, or exercise the investment to get the payoff $\Omega(x,t)$. Then the Bellman equation in (2.5) changes to

$$F(x,t) = \max\{\Omega(x,t), \pi(x,t)dt + \frac{1}{1+\rho dt}E_t[F(x+dx,t+dt)\}. \tag{2.7}$$

Applying Ito's Lemma and simplifying, we get the following well-known partial differential equation [DP94]

$$\frac{1}{2}\sigma^2 x^2 F_{xx}(x,t) + \alpha x F_x(x,t) + F_t(x,t) - \rho F(x,t) + \pi(x,t) = 0 \tag{2.8}$$

where the subscripts represent partial derivatives.

To solve for the equation, we need the two classic value-matching and smooth-pasting boundary conditions.

$$F(x^*(t),t) = \Omega(x^*(t),t) \tag{2.9}$$

$$F_x(x^*(t),t) = \Omega_x(x^*(t),t) \tag{2.10}$$

where $x^*(t)$ is the critical value on which point the investment is triggered.

The boundary condition in equation (2.9) says that if at $x^*(t)$ the investment is optimal, then the project value $F(x,t)$ should be equal to the termination value $\Omega(x,t)$. Equation (2.10) is known as the "high-order contact" [Sam65]. It implies that the values of $F(x,t)$ and $\Omega(x,t)$, as a function of x, should meet tangentially at the boundary $x^*(t)$ for the reason of maintaining continuity.

With these two boundary conditions, the value function $F(x,t)$ and the critical value $x^*(t)$ can be jointly solved.

Contingent Claims

Contingent claims analysis is based on the no-arbitrage theory in financial economics. If the cash flows of the invested project can be replicated by a portfolio of traded assets, the value of the project is then equal to the value of that portfolio. Otherwise, there is an arbitrage opportunity to make free money by buying the cheaper one of the two assets or portfolios, and selling the more valuable one. In asset pricing theory, the no-arbitrage condition requires that the risks in the state variable can be spanned by some traded assets.

In calculating the project value under this no-arbitrage assumption, the project must be operated optimally. Again, if this is not the case, one can buy the project at the price when it is not optimal and make a profit out of it. Thus, the optimal policies for the investment can be determined at the same time we obtain the value of the project.

Consider the same deferral investment option we analyzed with the dynamic programming method. Recall that this problem can be approximated by a binary decision problem at each time t. The firm either receives a cash flow $\pi(x,t)$ by waiting, or invest to get the payoff $\Omega(x,t)$. The investor's expected return comes from two sources. One part is the expected price appreciation α. The other part is the dividend (for stocks) or convenience yield (for commodities). Let $\upsilon = \alpha + \delta$ denote the total expected return, where δ is the continuous rate of dividend or the convenience yield. The risk free interest rate is r. Assume we invest one dollar in the risk-free asset and buy n units of the firm's output. This will cost $(1 + nx)$ dollars. In time dt, the risk-free asset pays a return rdt, while the other asset pays a dividend $nx\delta dt$ and has a capital gain of $ndx = n\alpha xdt + n\sigma xdz$. Then the total rate of return is

$$\frac{r + n(\alpha+\delta)x}{1+nx}dt + \frac{\sigma nx}{1+nx}dz \tag{2.11}$$

Compare this with holding ownership of the firm for the same interval dt. Let the value of the firm be $F(x,t)$, and this is the cost to buy the asset. The dividend is the profit $\pi(x,t)dt$. The capital gain of the asset can be calculated by using Ito's Lemma on $F(x,t)$, i.e.,

$$dF = [F_t(x,t) + \alpha xF_x(x,t) + \frac{1}{2}\sigma^2x^2F_{xx}(x,t)]dt + \sigma xF_x(x,t)dz \tag{2.12}$$

The total rate of return is

$$\frac{\pi(x,t) + F_t(x,t) + \alpha xF_x(x,t) + \frac{1}{2}\sigma^2x^2F_{xx}(x,t)}{F(x,t)}dt + \frac{\sigma xF_x(x,t)}{F(x,t)}dz \tag{2.13}$$

In order to replicate the risk of project, we have

$$\frac{nx}{1+nx} = \frac{xF_x(x,t)}{F(x,t)} \tag{2.14}$$

By the no-arbitrage principle, two assets with the same risk must have the same expected return. Thus the drift term in the two equations should be equal. This leads to the following PDE:

$$\frac{1}{2}\sigma^2 x^2 F_{xx}(x,t) + (r-\delta)xF_x(x,t) + F_t(x,t) - rF(x,t) + \pi(x,t) = 0. \quad (2.15)$$

subject to the same boundary conditions that are specified in equation (2.9) and (2.10).

Note that this PDE is very similar to the one we derived for dynamic programing method. A key difference between these two approaches is the treatment of the discount rate that decision makers use to value cash flows. In the dynamic programming approach the discount rate is chosen exogenously to reflect the opportunity cost of capital adjusted for the perceived risk of the particular project under consideration. Under the contingent claims approach only the risk-free rate of return is considered exogenous. The contingent claims approach assumes the existence of a sufficiently rich set of markets in risky assets so that the stochastic component of the risky project under consideration can be exactly replicated.

According to Dixit and Pindyck [DP94], the dynamic programing and contingent claims methods for real options should give the same results of valuation and critical investment thresholds, and the choice between these two methods is a matter of convenience. In all their examples where both methods are used, the results from both methods are exactly the same. However, when a constant discount rate is used, these two methods can only yield the same investment values under restrictive assumptions [IW06]. These assumptions require that discount rate in the dynamic programing method is exactly the risk-free interest rate, which implies that the market price of risk is equal to zero. The dynamic programing method is easier to incorporate operational constraints, but the usage of a subjective discount rate may lead to valuation result which deviates from the market price of the asset. The contingent claim method always gives the market price of the asset, but requires the existence of a sufficiently rich set of traded assets.

Black-Scholes

The Black-Scholes formula is a result of contingent claim analysis under strict assumptions. The six items in the first column of Table 2.1 are exactly the drivers of the option value in the Black-Scholes formula in equation (2.2).

Hence, as long as we can get appropriate estimates of the corresponding items in the second column for the real options [CA03], we can directly calculate the real option's value with the Black-Scholes formula [BS73], as if the real option is a financial option (see for example in [BE90] and [CA03]).

Due to its simplicity, the direct usage of Black-Scholes formula has gained some popularity among practitioners [CA03]. The problem lies in the imprecise nature of the analogy between financial and real options. Given the non-standard and non-financial aspect of real options, coupled with market incompleteness, the pricing of real options is more complicated. Even if we believe in the exact analogy between financial and real option by ignoring the limitations, the estimation of some of those items in the second column of Table 2.1 is always not an easy task.

2.5.2 Numerical Methods

Both the dynamic programing and contingent claims methods for real options reduce to solving a partial differential equation subject to certain boundary conditions. The closed-form solution to the partial differential equation, e.g., the Black-Scholes formula, rarely exists. In most cases, numerical methods, such as tree-building methods or simulations, are needed to approximate the solution to the partial differential equation.

Lattice/Tree Method

A binomial tree can be seen as a special case of dynamic programming, in which the decisions are binary. The binomial pricing model uses a "discrete-time framework" to trace the evolution of the option's key underlying variable via a binomial lattice (tree), for a given number of time steps between valuation date and option expiration. The state variable can either go up or go down by a specific multiplicative factor (u or d) in the next step of the tree. Following the classic Cox, Ross and Rubinstein (CRR) method [CRR79], we have

$$u = e^{\sigma\sqrt{\Delta t}} \quad (2.16)$$

$$d = e^{-\sigma\sqrt{\Delta t}} \quad (2.17)$$

where σ is the volatility of the underlying stock price and Δt is the time increment.

For a Brownian motion in the risk-neutral world, the probability for the state variable to go up, p, is given by

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{2.18}$$

where r is the risk-free interest rate.

At each final node of the tree, i.e., at expiration of the option, the option value is simply its intrinsic value, $Max[(S - K), 0]$, for a call option and $Max[(K - S), 0]$ for a put option. The option value at earlier nodes is calculated using the option values from the later two nodes (either up or down) weighted by their respective probabilities, p for up, and $(1 - p)$ for down. A recursive induction process with the following algorithm will work out the option value at the starting time point.

$$F_t = e^{-r\Delta t}[p \times \ F^u_{t+\Delta t} + (1 - p) \times \ F^d_{t+\Delta t}] \tag{2.19}$$

where F_t is the option price at time t, $F^u_{t+\Delta t}$ is the option value at time $t + \Delta t$ given the underlying price at time $t + \Delta t$ goes up by a rate of u from time t, $F^p_{t+\Delta t}$ is the option value at time $t + \Delta t$ given the underlying price at time $t + \Delta t$ goes down by a rate of d from time t.

As $\Delta t \to 0$, the binomial method result converges to the BS value [CRR79].

For American options, the decision needs to be made at each node choosing the immediate exercise or continuing to wait. In mathematics, the choice is determined by

$$\max\{F_t - K,\ e^{-r\Delta t}[p \times \ F^u_{t+\Delta t} + (1 - p) \times \ F^d_{t+\Delta t}]\} \tag{2.20}$$

Other lattice methods include trinomial trees [Boy86], adaptive mesh models [GAF99], etc. The lattice/tree approach is a flexible method and thus widely used to price both vanilla and some more exotic options. This approach is also relatively simple, mathematically, and can therefore be readily implemented in a computer program.

For options with several sources of uncertainty, or for options with complicated features, e.g., Asian options, lattice methods face several difficulties and are not practical. Valuation of options contingent on multiple factors becomes impractical due to the curse of dimensionality: the number of nodes of the lattice increases exponentially with the number of stochastic factors. In these cases, we can use the Monte Carlo simulation method. We will show this in the next subsection.

Monte Carlo Simulation

As a numerical method, Monte Carlo simulation has many inherent advantages such as its ease of accounting for more than one uncertainty, ease of incorporating different stochastic processes and non-standard payoff structure, etc.

The Monte Carlo simulation is a numerical integration technique that can be used to find a risk-neutral value of an option by sampling the range of integration [Boy77]. The fundamental theorem of no-arbitrage pricing tells us that the value of a derivative is equal to the discounted expected value of the derivative payoff under the risk-neutral measure. And an expectation is an integral with respect to the measure.

Thus let us suppose that our risk-neutral probability measure is Q, and that we have a derivative D whose payoff depends on a set of underlying instruments $S_1, S_{2,...}, S_n$. Given a sample ω from the probability space Ω, the value of the derivative is $D(S_1(\omega), S_2(\omega), ..., S_n(\omega)) =: D(\omega)$. The current value of the derivative is found by taking the expectation over all possible samples and discounting at the risk-free rate, i.e., the derivative has value

$$D_0 = df_T \int_\Omega D(\omega) dQ(\omega) \tag{2.21}$$

where df_T is the discount factor corresponding of the risk-free rate to the final maturity date T.

We approximate the integral by generating sample paths and then taking an average. Suppose we generate N samples, then we have a much easier calculation as

$$D_0 \approx df_T \frac{1}{N} \sum_{\omega \in \text{Sample set}} D(\omega) dQ(\omega) \tag{2.22}$$

Let us assume the underlying asset price follows a Geometric Brownian motion as in equation (2.1). To sample a path following this distribution, we divide the time interval $[0, T]$ into M units of length δt, and approximate the Brownian motion over the interval δt by a single normal variable of mean 0 and variance δt. This leads to a sample path of

$$S(k\delta t) = S((k-1)\delta t) \exp([(\mu - \frac{\sigma^2}{2})\delta t + \sigma\sqrt{\delta t}\varepsilon_k]) \tag{2.23}$$

where $k = 1, 2, ...M$, $\varepsilon_k \backsim N(0, 1)$ and is a random draw from a standard normal distribution.

Suppose the payoff function of one option on the underlying asset is $D(S,t)$. By generating N lots of M normal variables, we create N sample paths and so N values of D. Then the option value today given by the Monte Carlo method is

$$D_0 = df_T \frac{1}{N} \sum_{j=1}^{N} D_j(S,t) \tag{2.24}$$

Since we are able to keep track of the mean, maximum, minimum or any other statistics of the simulated paths, the Monte Carlo method is well suitable for valuing path-dependent options. Traditionally simulation was presented as a forward-looking technique, so it was seen as inadequate to deal with American options. In recent years, several researchers have proposed different methods to match simulation and dynamic programming, e.g., the Least Squares Monte Carlo (LSMC) method [LS01]. This enables the Monte Carlo method to price American and Bermudan options, which are frequent in capital budgeting projects. We will apply the LSMC method to the valuation of American investment options in Chapter 6.

The well-known drawback of the Monte Carlo method is its computational expense. The accuracy of the result grows with the number of simulations, and the required computational time grows exponentially with the dimension of the problem.

2.6 ROT Applications

Real options theory is an innovative tool in capital budgeting. It reveals the value of an investment project associated with uncertain market conditions and suggests the optimal investment strategies, e.g., the optimal timing, scale and technology. The real options valuation method, since its inception, has been proposed as an analytic tool for all types of investment problems — from natural-resource investments and new products to start-ups, acquisitions, factories, information technology, and more.

Real options found theirs use mostly in industries characterized by large capital investments, uncertainty, and flexibility — particularly oil and gas, mining, pharmaceuticals, and biotechnology [Tri96]. Companies in those industries also have plenty of the market or R&D data needed to make confident assumptions about uncertainties in real options analysis. In addition, they have the sort of engineering-oriented corporate culture that is not averse to using complex mathematical tools.

Earlier applications are focused on natural resources investment opportunities. Examples include oil fields, mines, forests (see for example [MS86], [PSS88], [BS85], [IW06]). We will discuss the real options applications in the energy sector in more detail in Chapter 3.

R&D investment is valued as options to open up future growth opportunities [CT99]. A vacant land in the real estate industry can be seen as an asset bearing options, since the owner can decide the timing and type of building for the development (see for example, [Tit85], [Qui93]). A forest is an option asset because the harvest schedule can be planned optimally to maximize its economic value [IW06]. The manufacturing flexibility to operate with different capacities is considered to be of significant option value (see for example [Kul88], [HP92] and [Bol99]).

Not only investment opportunities but also the capital structure of a firm can be analyzed with real options theory [MT94]. Trigeorgis applied real options to analyze credit risks that are encountered by financial institutes.

Strategic planning can be seen as a collection of real options [Lue98]. In addition to the optimal action rule required in realizing the option value, the managers can even take proactive measures under uncertainty conditions to improve option value by pulling one or more of the six levers in Table 2.1[6] [LM97].

ROT also applies to social life. Some interesting, although somewhat strange examples are the applications in assessing the waiting value in a marriage [Str03] and estimating the probability of suicide risk in the old age population [LK04].

2.7 Criticism and Defense of ROT

Real options theory has its root in the financial markets. However, the assumptions made for the financial markets may not be appropriate in other markets. This leads to criticism on the real options applications.

The first criticism on ROT comes from the doubt about the validity of the no-arbitrage pricing approach in real assets. In financial markets, the no-arbitrage pricing approach is based on the usage of portfolios of traded

[6] This leads to option-game analysis which integrates real options techniques and game theory. More details are given in [ST04].

securities to replicate the payoff of an option. Since most underlying assets in real-life investment projects are not tradable, the no-arbitrage principle seems to be losing its foundation.

Mason and Merton [MM85] argue that the justification of real options resembles the correctness of using NPV. A DCF analysis attempts to determine the value of an asset or a project as if it were to be traded. We identify for each project a twin security which has the same risk characteristics and is traded in the financial markets, and use the market required rate of return as the discount rate.

According to Trigeorgis [Tri96], the asset owner can, in principle, replicate the returns of a real option by a portfolio including shares of its twin security and risk-free bond. For the no-arbitrage principle to hold in a non-traded project, the option value must be the no-arbitrage value of the option on its twin traded security. The only adjustment needed is to reduce the equilibrium rate of return expected in the financial markets by a risk rate-of-return shortfall, a dividend-like adjustment. This is just the risk neutral valuation of the real assets.

Dixit and Pindyck [DP94] argue that the use of contingent claims requires the complete market assumption, i.e., stochastic changes in the underlying uncertainty must be spanned by existing assets in the economy. The assumption of spanning should hold for most commodities, which are typically traded on both spot and futures markets, and for manufactured goods to the extent that prices are correlated with the values of shares or portfolios.

The second criticism concerns the choice of a stochastic process for the underlying asset price. In a Black-Scholes setting, the underlying asset price is assumed to follow a continuous process. However, in a real asset, this assumption may be violated. For example, jumps may occur in prices. In this case, a deep-out-of-the-money option may be underestimated. A Geometric Brownian motion may not be a good approximation for the underlying. This problem can be overcome by employing more realistic models that implicitly account for the non-standard price distributions. For example, we can use a jump diffusion model, a regime switching model.

The third criticism concerns the exercise property of a real option. The exercise of a financial option is instantaneous, i.e., when the action is taken, the ownership transferred to the buyer. Real options cases are much more complicated. The exercise of a real option may involve the need to build a plant or to drill a bunch of wells. And these actions may take years to be completed. In this sense, the lifetime of some real options may be less than the stated life.

In some defending arguments against this criticism, real options model is thought to be able to factor in the technical need to incorporate the real exercise properties. In valuing the investment opportunity, the reduced lifetime adjustment are considered [AC06].

Finally, real options techniques are regarded, mainly by practitioners, as a "black box", due to the sophisticated mathematics, e.g., Partial Differential Equations, in real options, and the consequent lack of transparency and simplicity [Tea03]. But thanks to the increasing power of computers, commercial software vendors offer many user-friendly applications of complex real options.

2.8 Concluding Remarks

Real options theory takes into account explicitly the managerial flexibilities in assets or investment projects. Thus, it overcomes the shortfalls of the traditional Net Present Value method in determining asset value and optimal investment strategies.

Real options theory has its roots in the option pricing theories in the financial markets. The resemblance of the investment in real asset to a financial option validates the pricing technology to be transplanted in the real asset valuation and investment analysis. In consequence, both the analytical approximation and numerical methods can be employed to value real options.

Real options theory has been applied to almost all industries, and to almost all aspects of decision-making problems in academia. In practice, it has become both an important valuation method and a management tool.

With the deregulation of gas and electricity in many countries, the real options technique sees an increasing importance in the energy sector.

In the next chapter, we will review the various applications of real options to the energy markets.

Chapter 3

Real Options In The Energy Industry

3.1 Introduction

Real options theory has become a widely accepted tool for capital planning in many industries. Due to its inherent merit of explicitly accounting for managerial flexibility, real options methods outperform traditional discounted cash flow methods by giving more realistic valuation results and more reasonable investment suggestions.

The energy industry is a fertile field for real options applications. Early real options literature was mostly found in the natural resources projects, especially in the oil and gas upstream industry. A number of classic papers were written in this line (see for example in [PSS88] and [BS85]). The literature has been enriched by [PS93] and [Dia04]. The oil and gas upstream industry is characterized by its sequential investments. Real options are embedded along with the process. The option to wait is the most important type of real options in oil field investments, especially in undeveloped oil reserves. The oil price is regarded by most researchers as the underlying asset in valuing oil reserves. Different stochastic processes are used to model oil price dynamics. More complicated investment problems in oil field investment may include more than one type of real options and more than one type of uncertainty. Compound options and rainbow options have been used to address these issues.

One of the latest real options applications to the energy sector is in the electricity markets. The electricity markets have only been deregulated recently in the US, Europe and Australia. As a consequence, power companies are exposed not only to uncertain customer demand but also to fluctuating electricity spot and forward prices. Only the generating unit with the lowest marginal cost is called online to generate electricity. Thus the peak-load generators may operate only a small fraction of time. The owners of peak-load power plants turn on or shut down the units in response to the difference between electricity and fuel prices. This kind of flexibility in a power plant is named spark spread option, which will be discussed in more detail later in this chapter.

In addition to the spark spread option, there are many other flexibilities that are of significant relevance to a power company. A hydro power plant is known as a flexible asset with the possibility to manage the water level in its reservoir. Thus, for a hydro power plant, the operator can choose not only if it will run the turbine, but also when to generate electricity. As we discussed in Chapter 2, for an IGCC thermal power plant, the operator has the option to switch the fuels it burns, and for a CHP plant, the operator has the flexibility to adjust the mix of its outputs – heat and electricity.

Many other assets in the energy industry include such managerial flexibility and thus the corresponding real options. A gas storage facility can be viewed as a time spread option on the spot gas price. An LNG plant can be modeled as a spread option between using domestic gas and buying foreign gas [AC06]. Similarly, electricity transmission facilities can be thought of as a spread option, or a location switch option [DJS01] [Ron02].

Emission allowance is a recent area that real options can be applied to. The SO_2 trading system started in 1990. Electric utilities then have three alternative choices: they can either buy SO_2 allowances from the emission trading markets or retrofit the plant to accommodate low-sulfur coal, or invest in a "sulfur scrubber". The optimal decision is given by real options analysis [Her92]. The European Union Emission Trading Scheme started operation in January 2005 in compliance with the Kyoto Protocol requirements. During the first phase (2005-2007), the market participants receive the emission allowance for free. Thus power generation companies can choose either to produce electricity by "consuming" the emission rights or to stay idle and sell the emission allowance in the emission trading market.

The real options method aims at two related purposes: asset valuation and investment decision-making. The implications of real options are twofold: the short-term operation schedule of an asset and the long-term investment strategies. In this chapter, all these aspects are surveyed in different sectors in the energy markets.

The remainder of this chapter is organized as follows. Section 3.2 introduces real options modeling of oil and gas asset investments. Section 3.3 discusses the applications of real options in the electricity industry. Section 3.4 introduces the models for the valuation of gas storage facilities as spread options. In Section 3.5, we present some concluding remarks and discussions.

3.2 Oil and Gas Exploration and Production

The petroleum exploration and development rights are often transacted by government-created auctions. Market players always face the risk of paying too much for an acquired property, or selling for too little. The high transaction frequency and the huge investment amount in the petroleum property call for accurate and reliable valuation methods. Therefore, the ability to value petroleum properties accurately plays a critical role in determining the financial success or failure of oil and gas producers. Due to the various sources of uncertainty (both economic and geological), the valuation of oil and gas properties is not an easy task. In this section, we introduce how real options has been developed as an innovative technique to meet this challenge.

3.2.1 The Investment Phases and Relevant Real Options

The investment of oil and gas fields can be described as a sequential option process with sequential phases. These sequential real options together with investment decisions in each stage are illustrated in Figure 3.1.

During the exploration phases, the managers make a decision on undertaking wildcat drilling. The exploration expenditures are relatively small. So, in most cases, exercising the option to explore is optimal.

In case of success, i.e., oil or gas being discovered, the firm has the option to invest in delineation wells and additional 3D seismic tests, in order to get more information about the volume and quality of the reserves. These are the activities in the second phase – the appraisal phase.

If the information yields a promising prospect for the reserve, the firm then exercises its option to develop by committing a large investment in the develop phase. Otherwise, the firm can quit the undeveloped reserve by giving up the development right or wait until further favorable information. The option value, especially from the option to delay investment in this phase is important because of the huge capital expenditures involved.

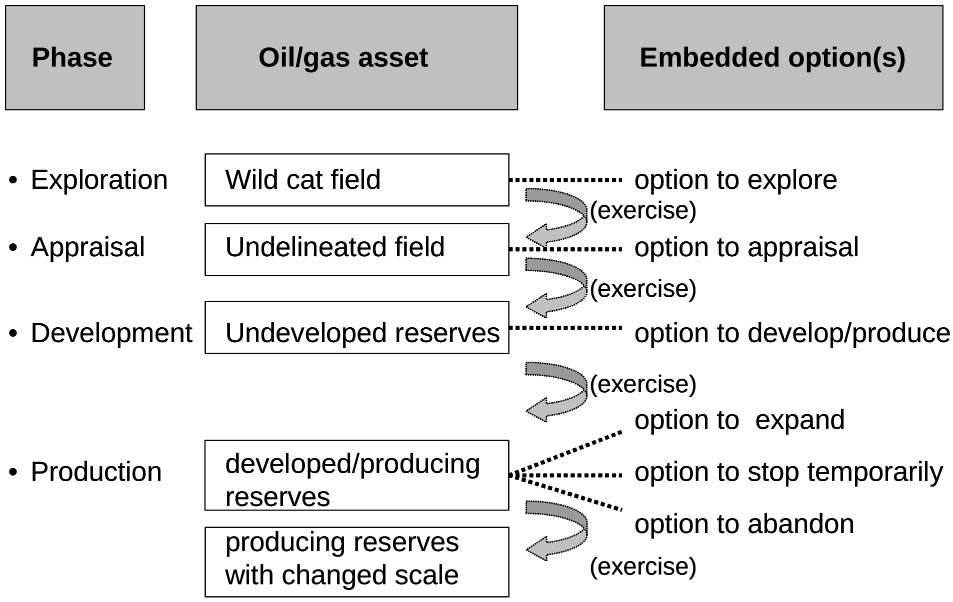

Figure 3.1: Oil and gas field real options

During the production life of the developed reserve, the firm has such operational options as the option to expand the production, the option of temporary suspension of the production, and the option to abandon the reserve.

The whole process can be viewed as a compound option, in which the options in a later phase depend on the exercise of the options on the previous phase. To simplify the problem, we often focus on only one specific type of option which is of particular significance in determining the investment opportunity value. In practice, the option to develop is often the choice. In doing so, we are just approximating the problem with an assumption that the other options in the project can be ignored[1].

[1] The value of the option to abandon a project is proved to be negligible in many investment cases [NF04].

3.2.2 Stochastic Processes for Oil Prices

The most important uncertainty in an investment is the price movements of the underlying asset. For producing reserves, the oil price is directly the underlying asset. For developed, or delineated reserves, the underlying asset is the value of a developed reserve, which in turn is a function of oil price. As an industry convention, a linear relationship is assumed between the developed reserve and the wellhead oil price (for example see [MS86] and [Dia04])[2].

The choice of the stochastic process for the modeling of oil prices is the starting point of the real options problem. For the same investment problem, different oil price processes may yield different valuation results and therefore imply different strategies. In the following context, we compare four different oil price processes that are used in literature. They are Geometric Brownian motion (GBM), Ornstein-Uhlenbeck, mean reverting jump diffusion, and two-factor models.

The GBM process is borrowed directly from the Black-Scholes framework in the financial markets. The underlying asset value follows a lognormal distribution with the variance growing with the forecasted time horizon, and a drift that grows (or decays) exponentially. This model was proposed by [PSS88] as a classical real options model for upstream petroleum applications[3]. Due to its simplicity and few parameters to estimate, this model has been popular for real options modeling.

GBM has proved an appropriate approximation for the behavior of equity prices. However, in the commodity markets, the mean reversion model seems to be more realistic since the balancing force from the demand and supply causes the market price to revert to a long-term equilibrium level (for more discussions see [BMP98] and [Pin99]).

In many cases, GBM is a good approximation for real options models (see for example in [Pin99], [Dia04] and [SS00]). However, GBM may be inadequate if the spot price is too far away from a more reasonable long-term equilibrium level. How far the spot price is differing from the long-term equilibrium can be roughly observed from the slope of the term structure.

In a GBM process, oil prices are totally unpredictable in that every change

[2]In practice, the price of one unit of developed oil reserve underground is traditionally set to be 33% of one unit of wellhead oil price.

[3]Note that in [PSS88], the underlying variable is the project value, not the oil price iteself.

in the oil price is a permanent shock in the long-term drift. To the opposite, the mean reversion process assumes every price deviation from the long-term equilibrium level to be temporary. The Ornstein-Uhlenbeck process is the most widely used mean reverting process in financial modeling. This process is used for oil prices as well.

These two extreme views on the price uncertainties are compromised in some other models. The mean reverting jump diffusion model for oil price was proposed by Dias and Rocha [DR98] by adding a jump term to the pure mean reversion model. The Poisson jump [Mer76] accounts for the modeling of the discontinuities in oil price. The jumps happen when there is abnormal news in the market, and this specification makes the model not "too predictable" as the pure mean reverting model.

Other important alternatives are the two-factor or three-factor models. The two-factor model was first introduced by Gibson and Schwartz [GS90], in which the oil price follows a GBM and the oil convenience yield follows a mean reverting process. In [Sch97], a two-factor model assumes both the logarithms of the oil price and the convenience yield are mean reverting. The three-factor model is built on the two-factor model by further assuming the interest rate to be stochastic. The two-factor or three-factor models are more realistic than the one-factor mean reverting model, since in the former models, the price movements are less predictable.

Schwartz and Smith [SS00] present another form of two-factor model. In their model the logarithm of the spot price[4] is assumed to be the sum of two stochastic components: the short-term price and the long-term equilibrium price. The short-term dynamics is driven by the short-term shocks and follows a mean reverting process. The long-term dynamics is determined by the changes in the fundamental market conditions and it follows a random walk. It is concluded that for many long-term investments, we may evaluate the investment by using the equilibrium prices only. This would reduce to a GBM model for oil spot price[5].

3.2.3 The ROV Framework

The contingent claim introduced in Chapter 2 is often used in oil or gas project valuation. Interpreting the investment project as a contingent claim,

[4] The spot price here is not the nearest futures price as a proxy for the spot, but a state variable derived from the term structure.

[5] [NF04] uses this conclusion in power plant investment decisions.

the project's market value must satisfy the fundamental stochastic partial differential equation. Thus, finding the critical threshold and project value with real option methods involves solving a sequence of PDEs, together with appropriate boundary conditions.

The Deferral Option

If we have a now-or-never investment opportunity, i.e., there is no option for investment timing, the threshold is then determined by equating NPV to zero, which will actually lead to the break-even point from traditional DCF analysis. On the other hand, if we have the option to delay the investment, we can solve for the threshold by equating the value of waiting and the value of immediate investment. The optimal investment rule is then to invest if NPV is equal or greater than the threshold NPV. Often, the threshold for the optimal option exercise, is the price of the underlying output commodity such as the price of developed reserve, or the oil price.

The option to delay is mostly considered in oil and gas field development decisions. The pioneering paper of Paddock et. al [PSS88] uses the value of one barrel of developed reserve, V_t, as the underlying variable, which follows a GBM given by

$$\frac{dV}{V} = (r - \delta)dt + \sigma dz \tag{3.1}$$

where r is the risk-adjusted rate of return, δ is the dividend-like payout rate, and σ is the volatility of the unit value of developed reserve, and dz is the increment of a standard Wiener process.

Denote by $F(V,t)$ the value of one unit of undeveloped reserve. Based on the contingent claim derivation in Chapter 2, $F(V,t)$ must satisfy

$$\frac{1}{2}\sigma^2 V^2 F_{VV}(V,t) + (r - \delta)VF_V(V,t) + F_t(V,t) - rF(V,t) = 0. \tag{3.2}$$

This equation is almost the same as equation (2.15) in Chapter 2, except that here we do not receive a cash flow by delaying the investment. So the term $\pi(V,t)$ disappears in this equation. Let D be the per-barrel development cost, the boundary conditions for the above PDE can then be written as

$$F(0,t) = 0 \tag{3.3}$$

$$F(V,T) = \max[V - D, 0] \tag{3.4}$$

$$F(V^*,t) = V^* - D \tag{3.5}$$

$$F_V(V^*,t) = 1 \tag{3.6}$$

The first condition says that when the underlying price is zero, so is the investment opportunity value. The second condition indicates that at expiration, the option to develop will be exercised if $V_T > D$. The third and fourth conditions are the value-matching and smooth-pasting conditions we already know in Chapter 2, in which V^* is the critical value of the developed reserve, i.e., if $V > V^*$, we should commit the investment immediately.

The closed-form solution for the above PDE only exists under special conditions. The closed-form solution only exists when (1) an infinite time horizon for the invest opportunity, i.e., $F_t = 0$, and (2) the project does not have an opportunity cost or dividend payout, i.e., $\pi(V, t) = 0$. If either of the two requirements is violated, we have to solve the PDE with numerical methods such as finite difference [BS78] [DP94], or binomial trees [PS93] [AC05].

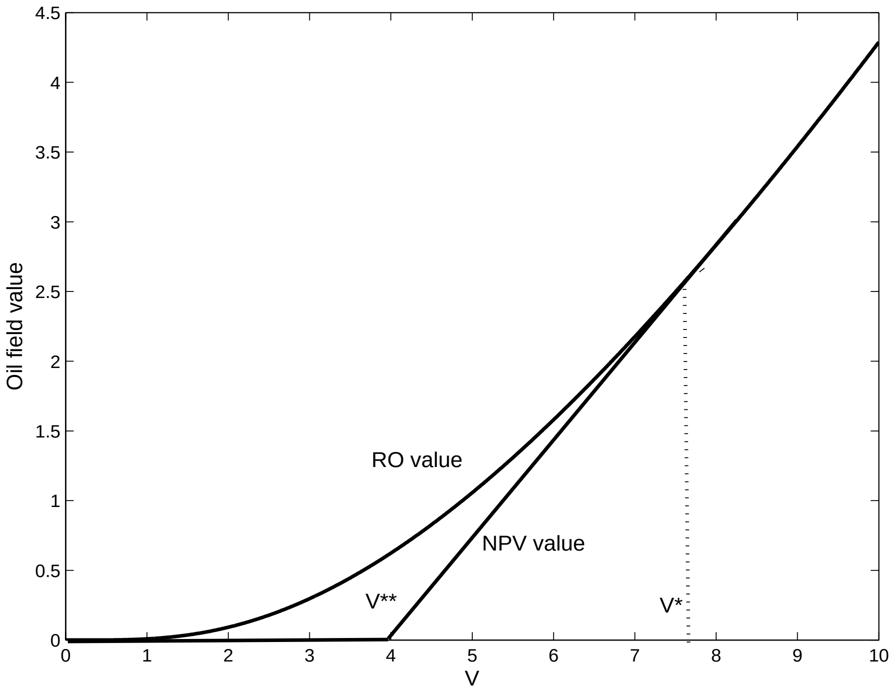

Figure 3.2: Undeveloped oil field value as a function of the unit value of developed reserve

The numerical examples in [PSS88] show that the critical value, V^*, is mostly significantly greater than the investment cost, D, as opposed to the

DCF investment critical value, where $V^{**} = D$. The result $V^* > V^{**}$ validates the value of waiting. In particular, before $V < V^*$, although the undeveloped reserve is not optimal to develop, its value of option to wait is still higher than the project value implied in the NPV method. Figure 3.2 illustrates these features.

Other Options

In order to the option to defer an investment, there are other flexibilities that can be incorporated in the real options analysis. We consider the option to temporarily shut down and the option to abandon.

Options to Shut Down. McDonald and Siegel [MS85] view the operating asset with an option to temporarily shut down as an option to acquire the project value V_t by paying the variable production cost C_t. Assuming the output price P follows a GBM process as defined in equation (2.1), i.e.,

$$\frac{dP}{P} = \alpha dt + \sigma dz \tag{3.7}$$

then the time t payoff is

$$\pi_t = \max(P_t - C_t, 0). \tag{3.8}$$

Further assuming $\delta = r - \alpha$ as the return shortfall in the output, and by applying the risk-neutral pricing process, the current value of a claim on the time t payoff is

$$\begin{aligned} F(P_t, C_t, t) &= e^{-rt} E^Q[\max(P_t - C_t, 0)] && (3.9) \\ &= V_0 e^{-\delta t} N(d_1) - C_t e^{-rt} N(d_2) && (3.10) \end{aligned}$$

where

$$\begin{aligned} d_1 &= \frac{\ln(P_0/C_t) + [(r-\delta) + \sigma^2/2]t}{\sigma\sqrt{t}} && (3.11) \\ d_2 &= d_1 - \sigma\sqrt{t}. && (3.12) \end{aligned}$$

Then the present value of the operating project with the option to stop temporarily is obtained by summing up the separate values of all such time t claims over the projects's lifetime, T :

$$R = \sum_{t=0}^{T} F(t) \tag{3.13}$$

Options to Abandon. The option to permanently abandon a project for a salvage value, S, is considered in Myers and Majd [MM90] as an American put option on a dividend-paying project. They assume the project value, V, under risk-neutral measure, follows a GBM process as defined in equation (3.1), i.e.,

$$\frac{dV}{V} = (\alpha - \delta)dt + \sigma dz \tag{3.14}$$

where δ represents the instantaneous cash payout from the project.

Then the value of the option to abandon, F, which is function of V and t, must satisfy the PDE as defined in equation (3.2), i.e.,

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + (r - \delta)VF_V - F_t(V, t) - rF = 0 \tag{3.15}$$

subject to

$$F(V, 0) = \max[S - V, 0] \tag{3.16}$$
$$F(0, t) = S \tag{3.17}$$
$$F(\infty, T) = 0 \tag{3.18}$$

At each time period, when the project value is below the critical value $V^*(t)$, the project should be given up. The option value, $F(V^*, t)$, is determined at the same time. As expected, the value of the abandonment option increases with salvage value, project volatility, and project lifetime, while it decreases with project value.

Compound Options

As we discussed in Section 3.2.1, the additional options, e.g., the option to abandon, the option to expand or to contract, and the option to temporarily stop a project, depend on the exercise of the development option, by which the producing oil field is acquired. Thus we often consider the additional options together with the main investment options. For example, if we have the opportunity to invest in an undeveloped field which includes an option to abandon, the option to develop the oil field is then an option on acquiring a developed field with abandonment possibilities. In such cases, the investment opportunity can be valued as compound options.

Geske [Ges79] derives a formula for valuing a European call option on another European call option on stock in the absence of dividend payouts.

This method is applied in [Er88] to value a satellite oil field as compound options.

Brennan and Schwartz [BS85] outline the framework to model additional options added to a main investment opportunity in a mine. They derive the differential equations governing the value of the mine by using the contingent claim method. They include not only the costs of opening and closing the mine, but also the tax and inflation rate in the economy. The derived PDE, together with the boundary conditions, suffice to determine not only the value of the mine[6], but also the optimal policies for opening, closing, and abandoning the mine.

Following the spirit of [BS85], Bjerksund and Ekern [BE90] revisit the compound options investment opportunity. With numerical examples, they find that the value of compound options is actually the value of main investment options plus a premium for the additional flexibility. The additional options lower the critical commodity price to commit investment. The option of temporally closing and reopening the mine has a larger effect than the option to abandon.

Integrating Market and Technical Uncertainties

As a special characteristic of the oil and gas industry, the investment made in the exploration stage as well as in the development stage can be viewed as a venture or a gamble. During the past 30 years, 72% of all exploration wells and 19% of all development wells in the US have resulted in "dry holes" [Smi03]. Many reasons can cause this kind of failure, such as the barren geological formations, deficiencies in the quality of the deposit that preclude production at a reasonable cost, and the technical failure or breakdown of drilling equipment.

In the traditional NPV framework, the dry hole risk can be directly incorporated by assigning a failure factor in the model. However, the failure possibility of a reserve is often not a constant, but actually a stochastic process. The normal approach to account for this additional uncertainty is to rewrite the developed reserve value into a function of other state variables.

Cortazar et al. [CSC01] define the mine value as a function of two state variables, the output price, P, and geological-technical risk, G. Thus the

[6] The initial status of the mine matters for mine value. Thus an open mine and a closed mine have different values.

mine value is actually $Z = PG$. Here G is assumed to follow a zero drift constant volatility Brownian motion as

$$\frac{dG}{G} = \sigma_G dw_G \tag{3.19}$$

and the output price P follows a GBM process under the risk-neutral measure.

$$\frac{dP}{P} = (r - \delta)dt + \sigma_P dw_P \tag{3.20}$$

The geological-technical risk factor is assumed to be independent of output price P[7], i.e.,

$$dw_p dw_G = 0 \tag{3.21}$$

Applying Ito's lemma, we have

$$\frac{dZ}{Z} = (r - \delta)dt + \sigma_P dw_P + \sigma_G dw_G \tag{3.22}$$

The new state variable, Z, can be seen as a modified commodity price with the same drift as P, but with an increased volatility

$$\sigma_Z = \sqrt{\sigma_P^2 + \sigma_G^2} \tag{3.23}$$

In this way, the joint price and geological-technical uncertainty are collapsed into a one-factor model. Following the [BS85] framework, we can value the investment option and additional operational options and obtain the optimal investment strategies.

An alternative way to simplify the problem of two uncertainties is to use a numeraire by calculating the ratio of the two state variables. This approach was first proposed by Margrabe [Mar78] in valuing an option to exchange one asset for another asset. Following the approach in [Mar78], Mcdonald and Siegel [MS86] study the investment option when both project output and investment cost are stochastic, Myers and Majd [MM90] investigate options to abandon with uncertainties in both the underlying asset price and the salvage value, Kulatilaka [Kul93] addresses the fuel switch option when two fuel prices are uncertain.

[7] Given modern development of oil recovery technologies, this assumption may be not true.

3.3 Electricity Supply Industry

As a consequence of the deregulation in the electricity supply industry, the traded electricity price provides the condition for pricing real options. In this section, we survey the main real options in the operation and investment of power generation and transmission assets.

3.3.1 Real Options in the Electricity Industry

A power plant can be viewed as a process that converts other types of energy – for example, fossil fuel, mechanical, wind, solar, tides – into electricity. In a competitive marketplace, owning a power plant can be viewed as holding the real options to execute this conversion process. The owner can at each time period decide whether to turn on the plant to produce electricity or not. With investment opportunities to build or acquire a power plant, the investor may have the possibility to wait until favorable market conditions emerge.

The option to generate electricity by burning a particular fuel is called the *spark spread option*. The spark spread is defined as the electricity price minus the product of the heat rate of the unit and the fuel price. The value of a thermal plant can be modeled as the sum of the values of a series of spark spread call options over the lifetime of the power plant (see for example in [DJS01], [GZ00], [TB02] and [Ron02]).

In addition to the spark spread option, a thermal power plant may have other operational flexibility. As we discussed in the previous context, an IGCC power plant has the option to switch the fuels during its operation. This option is named the *fuel switch option*. The value of the fuel switch option can be calculated following the method proposed by Kulatilaka [Kul93], in which the ratio of the two fuel prices is used as the new state variable to simplify the problem.

A *CHP plant* has the flexibility to adjust the mix of its outputs – heat and electricity, thus a spread option between the cost-adjusted electricity prices and the heat price can be used to value the plant. Similarly, electricity *transmission facilities* can be thought as a spread option between the electricity prices at two different locations [DJS01] [Ron02].

Electricity cannot be readily stored and consumers generally do not schedule increases or decreases in their consumption[8], so generators must be prepared to ramp up or down simultaneously with demand. Similarly, generators must stand ready to provide voltage or frequency support if power quality falls below operating tolerances. In regulated markets, spinning reserve[9] and other *ancillary services*[10] are included in the system capacity charge to customers. In competitive markets, generators can choose to offer ancillary service or to generate electricity. This is another type of spread option between the ancillary service price and spark spread option price.

Operators of *hydro power plants* must face more uncertainty such as the precipitation, and they may have more flexibility than running a thermal plant. The water in the reservoir is a storable commodity and can be transformed into electricity immediately. Thus the hydro plant operator may reveal more value by the management of the water level. And of course, this makes the hydro management a more complex problem. Nevertheless, some hydro plants also have pumps, with which water from a lower located lake is moved up to the reservoir.

Emission allowance is a recent area where real options can be applied to. The option value of emission allowance can be taken into account when valuing a power plant [Lau06].

Another operational option in a power plant is the *dynamic maintenance* of the unit [Ron02]. Since we have the option to decide the time to shut down the power plant for the required maintenance, we can maximize the power plant value under the price and load uncertainties.

We can also view the *contracted fuel* as an option, since we can choose either to generate electricity with the contracted fuel, or to sell the fuel to the market. In this approach, the contractual fuel is modeled as the spread option between the fuel price and the spark spread option price.

In order to limit the length of this thesis, we restrict our discussion in this subsection to a few important operational options, namely, spark spread option, transmission capacity, fuel switch options, and hydro power plants.

[8]The charateristics of electricity demand and suppy will be addressed in detail in Chapter 4.

[9]The spinning reserve is the extra generating capacity that is available by increasing the power output of generators that are already connected to the power system.

[10]Ancillary services are those functions performed by the equipment and people that generate, control, transmit, and distribute electricity to support the basic services of generating capacity, energy supply, and power delivery.

3.3.2 Spark Spread Option

The primary real option in a thermal power plant is the spark spread option. A spark spread option is based on the difference between the price of electricity and the price of a particular fuel used to generate it. It is an option that yields its holder the positive part of electricity price less the "strike" – heat rate adjusted fuel price[11], at its maturity time. For a rational power plant owner, the payoff obtainable at time t must be equal to the value of a properly structured spark spread call option.

The heat rate measures the efficiency of a generate unit. Traditionally, heat rate is defined as the number of British thermal units (Btu) of the input fuel required to generate 1 megawatt hour of electricity. From the definition, we know that, the lower the heat rate, the more efficient is the unit.

A European spark spread call option written on fuel G at a fixed heat rate K_H gives the option holder the right but not the obligation to pay K_H times the unit price of fuel G at the option's maturity time T and receive the price of 1 unit of electricity. Let S_E^T and S_G^T be the spot prices of electricity and fuel at time T, respectively. Denote the value of the option at time t by $C(S_E^T, S_G^T, K_H, t)$, then we have

$$C(S_E^T, S_G^T, K_H, T) = \max(S_E^T - K_H S_G^T, 0) \tag{3.24}$$

The cost-of-carry methods of constructing replicating portfolios for commodity derivatives cannot be applied to value electricity derivatives, because electricity is non-storable. Since electricity futures or forward is tradable, a method based on electricity futures or forward prices is used to fulfil the contingent claim analysis for electricity derivatives [DJS01] [Ron02].

The electricity and fuel futures/forward prices, F_E and F_G, are modeled by appropriate processes. Denote by $V(F_E, F_G, t)$ the value of a financial instrument that depends on the values of the electricity futures prices F_E, F_G and time t, then we can use the contingent claim analysis to derive a PDE subject to some boundary conditions. A closed-form solution, if it exists [DJS01], to this PDE, will be a function of $F_E^{t,T}$, $F_G^{t,T}$, K_H and $(T-t)$ and resemble the formula for exchange options in [Mar78]. Let us assume the European call option value has a form of $C(F_E^{t,T}, F_G^{t,T}, K_H, T-t)$.

Ignoring the operational characteristics of a power plant, the value of 1 unit of the time t capacity is then the value of the spark spread option at

[11]The production of heat rate and fuel price is also called the dispatch cost.

that time. Denote the virtual value of 1 unit of capacity of a power plant by V_{unit}, then V_{unit} is equal to 1 unit of the plant's time-t capacity over the remaining lifetime T of the power plant, i.e.,

$$V_{unit} = \int_0^T C(t)dt. \tag{3.25}$$

Using spark spread options to value power plants is based on several conditions. The most important one is that we must have continuous futures or forward curves for both electricity and the fuel. In practice, the lifetime of a power plant is much longer than the longest maturity time of futures contracts. Therefore, we must use extrapolation method to construct a full term structure [FL03].

A thermal power plant has many operational constraints, such as the startup or shutdown time and costs, minimum uptime, minimum off time, etc. These operational constraints have important effects on the option-based value. At the same time they complicate the power plant valuation problem to a large extent.

When the operational constraints are taken into account in valuing a power plant with a spark spread option model, additional dimensions are incorporated, the closed-form solutions derived from classical Black-Scholes models are not applicable any more, and numerical methods are called for. In literature, two methods are proposed to value such options: Monte Carlo simulation [TB02] [Ron02] and lattice method [DO03] [Ron02]. In Chapter 5 of this thesis, we will use Monte Carlo simulation to account for a variety of operational constraints as well as the load uncertainty of a power plant.

3.3.3 Transmission Capacity

Energy companies frequently exchange a commodity at one location for the same commodity at another location. In the case of electricity, the transmission lines, or grid, fulfill this mission[12].

In a real options framework, the transmission right between two locations, location 1 and location 2, is modelled as a spread option between the electricity prices of these two locations. A European call option on this location spread with maturity T gives its holder the right but not the obligation to

[12] Sometimes we use the term "wheeling" to describe the act of transporting electric power over transmission lines.

pay the price of 1 unit of electricity at location 1 at time T and receive the price of K_L units of electricity at location 2. K_L may be called the transmission efficiency factor. The transmission cost from location 1 and location 2 can be taken into account by setting K_L being less than 1[13].

Take the unit prices of electricity at the two locations as the two underlying variables, the location switch option, i.e., the transmission right, can be valued in the same way as in Subsection 3.3.2, where the spark spread option is valued. Again, when more operational characteristics are considered, numerical methods such as Monte Carlo simulation and lattice methods must be used to value these spread options [Ron02].

3.3.4 Operational Fuel Switch Option

The operational fuel switch option is defined as the flexibility for a power unit to switch between different fuels in generating electricity. Nowadays, industrial technology has enabled some power generating units to accommodate different fuels. For example, some IGCC plants can burn oil, coke, heavy refinery liquid fuels, natural gas, biomass, and urban solid waste [AC05].

With the operational fuel switch option, the choice of the fuel is determined by the cost difference of the fuels and the cost to switch from one fuel to another. Abstracting from many operational constraints, the generating unit should choose the fuel that minimizes the dispatch cost. The option value of the fuel switch is equivalent to the incremental cost saving of a flexible-fuel unit over the best of all the single-fuel units. Denote the value of the flexible-fuel unit as V_{flex}, the value of the single-fuel generation unit which can only burn fuel i as $V_i (i = 1, 2, ..., n)$, then the value of the fuel switch option, V_{FS} is calculated as

$$V_{FS} = V_{flex} - \max_{1 \leqslant i \leqslant n} V_i \tag{3.26}$$

For the sake of simplicity, we consider a base-load generation unit which can use two fuels: gas and oil. Then the two modes are "gas-fired" versus "oil-fired". Following the method used in [Kul93], we use $m = 1, 2$ to index the gas and oil mode respectively. The cost of switching from mode j to mode k is c_{jk}. For example, c_{12} is the cost of switching from gas to oil. If

[13] Electrical power is invariably partially lost during transmission, especially when within lower voltage lines.

the the unit continues operating in a mode, no switch cost will occur, i.e., $c_{mm} = 0$ for all m.

Following the spirit of [Mar78], the relative price of oil to gas, $P_t = P_{oil}/P_{gas}$, can be used to simplify the modeling. Given the high degree of substitution between these two fuel sources, it is reasonable to use a mean reverting process for P_t.

The revenue in each time period depends not only on the mode in which the unit operates, but also on the market price of P_t as well as time, t. Denote the revenue in the time interval $(t, t+\Delta t)$ as $\Pi(P_t, m, t)$. Let r be the risk-free interest rate. The present value of all future net profit flows given optimal behavior henceforth is denoted by $F(P_t, m, t)$. Optimal behavior means that the firm always chooses the current mode to maximize the present value of current plus discounted expected futures profits net of switching costs. This condition is given by a Bellman equation. Assume at time t the firm is in mode m, the equation is

$$F(P_t, m, t) = \max_l \{\Pi(P_t, l, t) - c_{ml} + e^{r\Delta t} E_t[F(P_{t+\Delta t}, l, t + \Delta t)]\} \qquad (3.27)$$

where $l, m = 1, 2$ and $t = 0, ..., T - \Delta t$. The firm in each period chooses the mode, l, that maximizes the value of the generating unit.

With the calibrated model for P_t, the above Bellman equation can be solved with numerical methods [Kul93]. Then we have $V_{flex} = F(P_0, 1, 0)$ or $F(P_0, 2, 0)$, given the starting fuel is gas or oil. We can solve for the value of the single-fuel generation units, V_i, in the same way. Thus by equation (3.26) we obtain the value of the fuel switch option, V_{FS}.

Monte Carlo simulations can also be chosen to value fuel switch options, especially when we incorporate more operational constraints.

3.3.5 Hydro Power Plant Flexibilities

A hydro plant must have a reservoir, in which water is stored. By utilizing the difference in altitude between the reservoir and the turbines, this potential energy of water is converted into mechanical energy by letting the running water propel the turbines. In the generators, this mechanical energy is converted into electrical energy.

A hydro plant has more flexibilities than a thermal plant, because water is a storable commodity and can be transferred into electricity immediately. The operator of a hydro plant may reveal more value by the management of the water level.

Some of the hydro storage plants also have pumps. And if pumps are installed the owner also has the additional option to pump up water back to the reservoir, i.e., to convert electrical energy back into potential energy. This fact makes it possible not only to participate at high prices, i.e., turbining at maximum when the spot price is high, but also to make profit from lower prices by pumping water upwards in order to increase the amount in the upper lake that can be used later on.

The stochastic uncertainties we face in a hydro plant are, according to [FWZ02], electricity prices and inflows to reservoirs. The reservoirs store water in a storage dam and new water is floating into the dam from precipitation and melting snow. This inflow is random and seasonal.

The hydro storage plant has certain limits in its operation, depending on the possibility to change the level of stored water. Normally, a maximum and a minimum water level are specified for a certain reservoir. And due to ecological and legal reasons, the spill, or overflow at each time period is restricted to a certain level. Moreover, there are technical restrictions for both turbining and pumping, a minimum and a maximum amount of water that can be handled at each time period.

The marginal cost to produce electricity from a hydro storage plant is very low, since production is not subject to any fuel costs. The marginal cost is low enough to motivate selling base-load power. The owner of such a plant consequently has the option in each period to produce at a marginal cost. With the low marginal cost, one would like to produce in every period. However, on the other hand, since the stored water is limited, by exercising an option the water level decreases, the probability that an option in the future can be exercised decreases. If there is a pump, by pumping up water, this probability increases as the water level rises. The decision made today about producing and pumping, will thus affect the power plant's dispatch possibilities in the futures. An owner of such a plant would try to sell this power during peak hours to maximize the profit.

Hydro operation thus involves a continuous process of deciding whether to release water now or to store it and release it later on. When there is no pump, the decision variables for the dispatch are reservoir discharge, spill and reservoir level [FWZ02]. When a pump is installed, the decision variables are the turbining and pumping amount in each period (see for example in [Ung02], [TDR04] and [Doe06]).

The hydropower scheduling problem is modeled in linear stochastic programing, in which we seek the strategy to maximize expected cash flows subject to the appropriate restrictions. [TDR04] works in the continuous time and derives the PDE for the power plant value. This PDE, subject to the boundary conditions, are solved with finite difference method. The value of the power plant and the optimal strategy at any time are obtained simultaneously.

[FWZ02], [Ung02] and [Doe06] model hydro power plants with linear stochastic programing within discrete time periods. In all these articles, the hydro scheduling is modeled as a portfolio optimization problem, in which the possibility of trading financial contracts is incorporated. In linear programing, the producers' expected profit subject to a risk constraint is maximized under the uncertainties of electricity spot prices, water inflows and financial power contract prices.

The value of water in the hydro dam can be divided into three components which build up the total value [Ung02]. The first component is the static value that would be achieved by producing a constant amount without an active strategy and equals the average electricity spot price. The second component is the value stemming from the dynamic strategy resulting in a price of produced electricity that differs from the average spot price. The last component is the hedging value of the water, allowing us to take riskier positions in the contract portfolio. The hedging value of a hydro plant is supported by [Doe06], which finds out that utilities with a high degree of flexibility should be valued within a portfolio. When being valuated as stand-alone assets, these utilities would be underpriced in the market.

3.3.6 Investment Real Options

So far in this section, we have focused our discussion on the operational flexibility in electricity assets. With these operational options embedded, the electricity assets, for example, power plants, are assumed to operate with the optimal strategies in order to realize their option-based values. In operational options modeling, we were in fact assuming that once the power plant is in place, it will be run with the same configuration until the end of its lifetime. Moreover, we have assumed that if a plan to build a new power plant is approved, the new plant will be built immediately. We will relax these assumptions in this subsection when investigating the investment real options.

In the context of capital planning, we go back to the classic real options models as we discussed in Chapter 2. Namely, in power plant investments, we may have the option to wait, the option to abandon, the option to contract (scale back a project), the option to expand or upgrade the invested power plant, etc. In addition, we may have the option to choose among different power plant technologies.

The option to abandon a power plant has a negligible value [NF04b]. In contrast, the option to wait, the option to expand or upgrade the power plant and the option to choose a best technology are of significant relevance. In this subsection, we will investigate the option to expand or upgrade and some technology selecting options.

Classic Investment Options

After a licence is approved for the investment of a power plant, there is often a possibility to wait some time to execute the investment. In such an investment, the American type of option to wait is relevant. Until the expiration of the licence, we need to determine the optimal time to invest.

[AC05] use a one-factor model to value the option to invest in a power plant by assuming an exogenous electricity price. The stochastic gas price, S_t, is modeled by an inhomogeneous GBM (IGBM) process. The IGBM process will be discussed in Chapter 4.

The opportunity to invest in a power plant is modeled as an American call option on an operating power plant. The net present value of an operating power plant, $V(S_{t,}t)$, is a function of the current gas price.

A binomial tree is constructed in [AC05] to price the investment options. Denote $W(t)$ the option value at time t. Then the decision at each time t is to choose the maximum between the value of investing and the value of waiting. In a binomial lattice, we have

$$W(t) = \max[V(S_t, t), e^{-r\Delta t}(p_u W^+ + p_d W^-)] \tag{3.28}$$

where r is the risk-free interest rate, Δt is the time interval in each step in the binomial lattice, p_u (p_d) is the possibility that the gas price goes up (down) in next step, and W^+ (W^-) is the option value of the next step if the gas price goes up (down).

The binomial tree can be calculated backwards from the maturity until the time 0 value of the option is obtained. If we compare this value with that of an investment made at the outset, the difference will be the value of the option to wait. By changing the initial value for the fuel cost, it is possible to determine the fuel price at which the option value changes from positive to zero. This will be the optimal exercise price for the investment.

As we discussed earlier in Subsection 3.3.2, the spark spread option is the primary variable in determining the power plant value. Modeling the spark spread involves at least two uncertainty sources – electricity price and fuel price. One way to simplify the problem is to take the spark spread as a single variable [NF04]. Although in [NF04], a two-factor model following the spirit of [SS00] is used for the spark spread, the investment decision is assumed to be determined only by the equilibrium price, and then the problem reduces to the traditional one-factor real options modeling [DP04]. In Chapter 6, we will give more discussions on the investment options using spark spread as the single state variable.

In a long-term horizon, we may also have the possibility to upgrade or expand a power plant. The option to invest in an upgradeable base-load power plant is addressed in [NF04]. With a certain cost, the base-load power plant can be upgraded into a more efficient peak-load power plant. Denote F_u as the value of the option to upgrade, V_B as the base-load power plant value, V_P as the peak-load power plant value, we have

$$F_u = V_P - V_B \tag{3.29}$$

[AC06] investigate the option to double the size of a power plant within a finite time period. In valuing this expansion option, the gas price is following a two-factor IGBM model as in [AC05], and the electricity price follows a one-factor IGBM model. The Least Square Monte Carlo (LSMC) approach proposed by [LS01] is employed to price the American call option in a five year period. In the numerical example, it is shown that the project of investing in an expandable power plant has a much higher value than the value of investing initially in two smaller plants.

The option to abandon in a power plant investment has been studied in [NF04], where the possibility to abandon a power plant is considered as an additional flexibility. It is shown that the abandonment does not change the thresholds of the option to build the power plant. Thus, the power plant investment decision can be made by ignoring the abandonment option.

Technology Choice Decisions

Under output or input price uncertainties, the choices between different technologies can be modeled as real options as well. Here, we introduce three case studies on these investment problems.

In Subsection 3.3.4, we have discussed the operational fuel switch options. One property of these options is that their lifetime is normally very short, mostly a few hours. If at a certain time, we have chosen to switch from one certain fuel to another, in next few hours, we can choose to switch back again. In the long term, the power plant may change from burning one fuel to another, permanently. For instance, we can alter a coal-fired plant into an oil- or gas-fired plant by committing a capital investment. We can call this option the permanent fuel switch option. Opposite to the operational fuel switch, a permanent fuel switch option is an investment problem, where a capital investment is involved, and a permanent fuel switch option is economically irreversible.

The permanent fuel switch is an American type option, which means the owner can execute the investment to change the fuel at any time during a certain time period.

[Her92] applies the real options method to an existing coal-fired power plant that is required to comply with the new SO_2 emission limits introduced by the Clean Air Act Amendments of 1990 in the U.S. By assumption, the power plant operator can either purchase emission allowances from other utilities, or switch fuels to a lower-sulfur coal, or install an SO_2 emission reduction system. The two state variables considered are the future price of SO_2 allowances, and the price premium per unit of low-sulfur coal versus high-sulfur.

With a contingent claim analysis, the value of the fuel switch option can be obtained by solving a derived PDE with regard to the two state variables. Let F_{SW} be the value of the fuel switch, F_{SCR} be the value of the option to install scrubbers and $E(t)$ be the emission cost at time t, then the decision rule for choosing the technology is

$$\min[E(t), F_{SW}(t), F_{SCR}(t)]. \tag{3.30}$$

[NF04] consider the technology choice between an upgradeable base-load power plant and a peak-load power plant. Along with the valuation of the upgrade option, the critical equilibrium spark spread price, i.e., the upgrade threshold, ξ_{H1} is obtained. Then the choice of optimal technology is determined by comparing the current equilibrium spark spread price, ξ_0, and ξ_{H1}. If $\xi_{H1} < \xi_0$, we choose the upgradeable base-load power plant; if $\xi_{H1} \geqslant \xi_0$, we choose the peak-load power plant.

[AC05] study the value of an option to invest either in a NGCC[14] or a IGCC[15] power plant. At each moment, the choice can be made among: (1), to invest in the inflexible technology NGCC; (2), to invest in the flexible technology IGCC; or (3), to wait and at the maturity give up the investment. The problem is again modeled on a binomial lattice. At the final date, the investor should choose the best alternative among the three choices. The investment option value W at final date is then given by

$$W = \max(V_{igcc}, V_{ngcc}, 0) \tag{3.31}$$

where V_{igcc} and V_{ngcc} are the value of an IGCC and an NGCC power plant, respectively.

Since an IGCC has the additional flexibility to accommodate coal as its fuel, another state variable – the coal price – is added to the model in valuing an IGCC power plant. The binomial lattice then has two dimensions. At previous moments, the option value W must satisfy

$$W = \max[V_{igcc}, V_{ngcc}, e^{-r\Delta t}(p_{uu}W^{++}+p_{ud}W^{+-}+p_{du}W^{-+}+p_{dd}W^{--})] \tag{3.32}$$

where V_{igcc} and V_{ngcc} are defined in the same way as in the equation above, r is the risk-free interest rate, Δt is the time interval in each step in the binomial lattice, p_{uu} (p_{dd}) is the possibility that both the gas and coal price goes up (down) in next step, and W^{++} (W^{--}) is the option value of the next step if both the gas price goes up (down), p_{ud} is the possibility that the gas price goes up and coal price goes down in next step, W^{+-} is the option value of the next step if the gas price goes up and coal price goes down, p_{du} is the possibility that the gas price goes down and coal price goes up in next step, W^{-+} is the option value of the next step if the gas price goes down and coal price goes up.

Working backwards on the lattice until time 0, the initial investment option value can be obtained. At time 0, if $W = V_{igcc}$, the NGCC technology will be adopted; if $W = V_{ngcc}$, we should choose the IGCC technology; if $W = 0$, the best decision at time 0 is then to wait.

[14]NGCC stands for a natural gas fired combined cycle .

[15]IGCC stands for an integrated gasification combined cycle.

3.4 Gas Storage Facility

Gas storage facilities are built in order to bring gas as close as possible to the markets served and to maximize the availability during the high demand months. When the demand is low, such as in summer time in Europe, gas is injected into storage. When the demand is high, such as in winter time, gas is withdrawn to meet peak demand.

Gas storage facilities add value in two ways. Firstly, they are thought of as an arbitrage mechanism that allows the owner to make profit out of time spread, i.e., by buying gas in low-price months and selling gas in high-price months. Secondly, they are assets with inherent operational flexibility.

The value of a gas storage facility can be regarded as the maximum expected revenues that the operator of the facility can obtain by optimally operating the facility. In consequence, the option-based valuation lies in the optimal injection and withdrawal strategies that capture favorable spreads between the spot and the forward markets.

The option-based valuation of gas storage facilities is characterized as a stochastic control problem. The state variable that describes the market uncertainty is the gas price. A forward curve is able to determine the seasonal price spreads. The control variable is the injection or withdrawal volume at each time period. Moreover, just like a power plant, a gas storage has a bunch operational constraints. The injection and withdrawal rates determine how fast gas can be injected into or released from the storage. The higher these rates are, the more flexible the gas storage is. Gas injection and withdrawal also involve some fuel consumption and variable costs.

Two contrasting approaches have been used to solve this stochastic control problem numerically: PDE-based approaches and simulation-based methods. If we assume that the control only takes values from a finite set, simulation-based methods can be used to solve the stochastic control problem [Ron02]. If we have to approximate the control as piecewise constant, the PDE method is needed [TDR03]. The latter method is introduced below.

The current price per unit of natural gas, P, is chosen as the state variable and is assumed to follow a jump diffusion process as

$$dP = \mu dt + \sigma dX + dq \tag{3.33}$$

where μ is the expected rate of return on gas prices, σ is the volatility of gas prices, dX denotes the increment of a standard Brownian motion, and dq is a Poisson process with a jump size of γ and a jump intensity of λ.

Under a risk-neutral measure, a gas storage valuation is obtained by maximizing its expected cash flows over its lifetime T [TDR03] [Gem05]. The optimization problem is described as

$$\max_{c(P,I,t)} E[\int_0^T e^{-r\tau}(c - a(I,c))Pd\tau] \tag{3.34}$$

subject to

$$\begin{aligned} c_{\min}(I) &\leqslant c \leqslant c_{\max}(I) && (3.35) \\ I_{\min} &\leqslant I \leqslant I_{\max} && (3.36) \\ dI &= -(c + a(I,c))dt && (3.37) \end{aligned}$$

where c represents the amount of gas currently being released from ($c > 0$) or injected into ($c < 0$) storage, I is the current amount of working gas inventory, $a(I,c)$ is the amount of gas that is lost given that c units of gas are being released from or injected into storage and I units are currently in storage, $c_{\max}$ is the maximum delivery rate, i.e., the maximum rate at which gas can be released from storage as a function of inventory levels, $c_{\min}$ is the maximum injection rate, i.e., the maximum rate at which gas can be released from storage as a function of inventory levels, $I_{\max}$ is the maximum storage capacity of the facility, $I_{\min}$ is the minimum storage capacity.

Denote $V(P,I,t)$ as the corresponding option value of the gas storage determined by equation (3.34). Employing Ito's lemma on gas price P and gas in storage I, we can derive a PDE with appropriate boundary conditions to obtain the optimal strategy $c(P,I,t)$ and corresponding optimal value $V(P,I,t)$.

Generally, if there is no gas in reserve, nothing can be released, the gas storage unit is essentially a put option on gas price. To the opposite, the gas storage unit at high reservoir levels is essentially a call option on gas prices. For inventory levels somewhere between maximum and minimum capacity, the gas storage facility is like a financial straddle with both put and call properties.

3.5 Concluding Remarks

Real options have been intensively applied to the energy industry. Many energy assets, such as undeveloped oil fields, power plants, power transmission lines, gas storage facilities, etc., can be modeled as options.

The real options valuation of an energy asset involves optimizing the operation of the energy asset over its lifetime. Under market uncertainties which are represented by the dynamics of the state variables, the optimal control variables, such as the operational mode (on or off), the volume of energy release or injection, are determined by solving the PDEs that are derived from either a dynamic programming or a contingent claim analysis.

The real options valuation of energy assets can not only reveal the value of the operational flexibility which is ignored by DCF methods, but also suggest the optimal operation schedules under current market conditions. This managerial implications provide the valuable guidelines for the daily operations of energy assets.

Taking into account the flexibility of energy assets and investment, the option to invest in energy assets is actually a compound option on the option-embedded flexible assets. In an energy investment, the option to wait, the option to expand or upgrade may bring significant values.

The evolution of the market scenarios plays an essential role in determining the option-based asset values and the optimal strategies. In the next chapter, we will investigate the deregulated electricity markets, and empirically test the electricity models in two European markets.

In the next chapter, we will survey different pricing models that are used in deregulated markets. An empirical testing of prices models is also to be done with the Dutch and German market data.

Chapter 4

Electricity Prices Modeling

4.1 Electricity Markets

In this section, we present the background of electricity industry deregulation, with a special attention to the European countries. We also introduce how the markets are organized after deregulation.

4.1.1 Electricity Industry Deregulation

Electric power, often known as power or electricity, involves the production and delivery of electrical energy. Once it is generated, whether by burning fossil fuels, harnessing wind, solar, or hydro energy, or through nuclear fission, it is sent through high-voltage, high-capacity transmission lines to the local regions in which the electricity will be consumed. When the electricity arrives in local regions, it is transformed to a lower voltage and sent through a local distribution network to consumers.

Electricity was long considered a textbook example of natural monopoly. Governments were involved heavily in the electricity sector either as owner or as regulator. Traditionally, both the planning of the electric system and its operations have been the responsibility of regulated integrated utilities that generated, transmitted and distributed electricity. It was widely accepted that governments were the best choice to mobilize the large amounts of capital necessary to develop the sector and bear the long time horizon for recovery of costs. Particularly in developing countries, government leadership in the development and use of electricity was part of a broader "social compact".

In the past few decades, the electricity supply industry in many countries has transited from vertically integrated monopolies toward a competition-oriented market. Several forces stimulated the wave of electricity restructuring. Among them, the main driving forces are the global trends in the development of ideology, financing, and technological change [Jos03]. In mature industrial economies, the pressure for changes came from the emergence of excess capacity and the inefficiency of electricity sectors. In developing countries, the main driving force is the financing gap. The development of new generation technology, such as the combined cycle gas turbine (CCGT) has greatly reduced the minimum efficient scale of a generating plant, allowing flexible investment and operation of generators. Finally, the restructuring and privatization of the electricity sector has occurred in the context of a wholesale privatization of several other state-owned industries. The experience of privatization in other industries, especially in the gas industry, encouraged the restructuring of electricity sector.

The earliest introduction of market concepts and privatization to electric power systems took place in Chile in the late 1970s. Argentina improved on the Chilean model by imposing strict limits on market concentration. The deregulation wave spread over other Latin American countries, including Peru, Brazil and Colombia. However, deregulation in these countries gained limited success.

The privatization of the electricity supply industry of UK in 1990 is viewed as a milestone in the world electricity markets. The process was followed by Australia, New Zealand, and regional markets such as Alberta in Canada. The UK and the Nordic countries are viewed as the flagships of the reforms boasting lower prices, and have maintained supply security and service standards.

In contrast, in the US, in the aftermath of the California electricity crises in 2000 and 2001, the restructuring process has slowed down significantly and many states have put their reform plans on hold.

In the European Union, the first Electricity Directive (1996/92/EC) was passed in 1996, representing an important step toward market opening in electricity supply industry. The EU member states were required to adopt the directive by 1999. The new Electricity Directive (2003/54/EC) agreed in 2003 requires that all non-household customers can freely choose their electricity supplier by 1 July 2004, followed by full market opening to include all household customers by 1 July 2007. Against the background of a world-wide slow-down in the pace of electricity reform, the centrally driven effort by the European Commission has been the main force keeping the program on course [JP05].

In different deregulation processes, the market designs and institutions differ from each other, but many of the underlying concepts are the same. The common process involves separating the electricity generating and retail functions, which are contestable, from the natural monopoly functions of transmission and distribution. Meanwhile, a wholesale electricity market and a retail electricity market are established, and a controlling agency, i.e., the power system operator, is authorized to coordinate the dispatch of generating units to meet the expected demand of the system across the transmission grid.

4.1.2 Electricity Trading

An electricity market is a system to carry out the purchase and sale of electricity, where the market price is determined by matching electricity supply and demand. As a consequence of deregulation, electricity trading has grown dramatically and numerous power exchanges have emerged[1]. Electricity is defined as a commodity, since electron cannot be differentiated. However, this "commodization" of electricity applied mainly at the wholesale level.

The role of the wholesale market is to allow trading between generators, retailers and other financial intermediaries. Wholesale transactions in the physical commodity are typically cleared and settled by the grid operator.

In the electricity wholesale market, many types of contracts have been developed. These contracts can either be sold on the bilateral market, which is also named the Over-The-Counter (OTC) market, or on an organized power exchange. Electricity contracts can also be categorized by their settlement method into physical contracts which are aimed for delivery, and financial contracts, which are settled in cash. All electricity contracts share three characteristics: a defined delivery period, a certain amount of electricity, and a price.

Due to the high transaction cost of spot electricity trading, spot markets are usually organized by a power exchange. In most European countries, a power exchange is a voluntary marketplace in competition with the classic bilateral market. A power exchange provides a spot market (mainly day-ahead) for electricity transactions on the electricity to be delivered on the

[1] In Europe, the main power exchanges include Nord Pool (Norway, Sweden, Denmark, and Finland), EEX (Germany), Powernext (France), APX (Netherlands, Belgium, and UK), Endex (Netherlands), Belpex (Belgium), OMEL (Spain), and GME (Italy).

next day. Participants submit their purchase or sale orders electronically. The supply and demand are aggregated and compared in the power exchange to decide the market price for each hour of the following day.

In addition to the spot markets, the derivative markets are also organized to manage the financial risk associated with electricity price volatility. Derivatives contracts traded in power exchanges include forward contracts, futures, swaps and options. In the OTC markets, more exotic contracts, such as swing options, bulk forward contracts, etc., are traded.

4.2 Behavior of Electricity Prices

Certain features of electricity prices distinguish electricity markets from financial markets and, to a lesser degree, from most other commodity markets. In this section, we examine the characteristics of electricity prices and discuss the reasons behind these features.

4.2.1 Nonstorability

Many types of energy can be stored under different forms, while electricity cannot be stored economically once generated. Electricity is extremely costly to store. The technologies for storage, for instance hydroelectric pump storage or batteries are quite inefficient. Combined with the high cost of storage is the need to balance supply and demand second by second. A shortfall or surplus of electricity can endanger the stability of the entire electricity grid.

Moreover, almost no end-use consumers of electricity even have the technology to observe, let alone respond to, real-time prices. Electricity demand, especially the demand of residential users, is extremely inelastic in the short run. Thus, little of the real time balancing can be done on the demand side.

The nonstorability of electricity makes it a very special commodity. Since it is impossible to smooth out changes in production capacity availability or changes in demand by means of storage, electricity needs to be produced and sold "at the speed of light". Meanwhile, the cost of electricity – and therefore its price – varies from moment to moment.

4.2.2 Supply Stack

In a deregulated electricity market, the spot price is determined by matching aggregated asks and bids for a specific time period. All the bids from the market are arranged in a "supply stack". Knowing the characteristics of the different plants in a given region, one can build the supply function by stacking the units in "merit order", from the lowest to the highest cost of production. Thus the prevailing electricity price will be the marginal cost of the last unit that is called on to the grid. The logic behind the supply stack is that a rational operator would first dispatch the unit with the lowest marginal cost followed by units with higher marginal cost and so forth. This is consistent with the motive for introducing competition to electricity markets. In Figure 4.1 the solid line gives an example of a supply stack.

On the left side of the supply stack are plants that have very high capital and operating costs and very low variable (fuel) costs. These plants should be the first to be run and they generally run at full capacity all the time, forming the base load. These "must-run" plants include nuclear units, some steam-based fossil fuel plants, large coal-fired plants, and run-of-river hydro units. Since the marginal cost – mainly the fuel cost – is very low once the plant is up and running, the supply stack resembles a horizontal line for these generation units. At the right end of the supply stack are the peak units, which have the highest variable (fuel) costs. These plants are only turned on under temporarily high demand conditions after other plants are being used. At off-peak times, the peak units are conceived as reserve capacity. These peak units may include some gas-, oil- or diesel-fired turbines. Between the base- and peak-load units, the supply stack is constructed by mid-merit units. The variable costs and capital costs of mid-merit units are in the middle range.

As a whole, the electricity supply stack turns out to be a convex curve with a long, flat shape on the left side rising in a steep slope to the peak units on the right. The nearly vertical curve for the peak units reflects a tight market condition when the demand approaches the total available capacity.

The dashed vertical line in Figure 4.1 represents the demand curve. The spot price is determined by the intersection of the aggregate demand and supply functions. A forced outage of a major power plant or a sudden surge of demand due to extreme weather conditions would either shift the supply curve to the left or drive the demand curve to the right, in both cases the spot price can rise to an extremely high level.

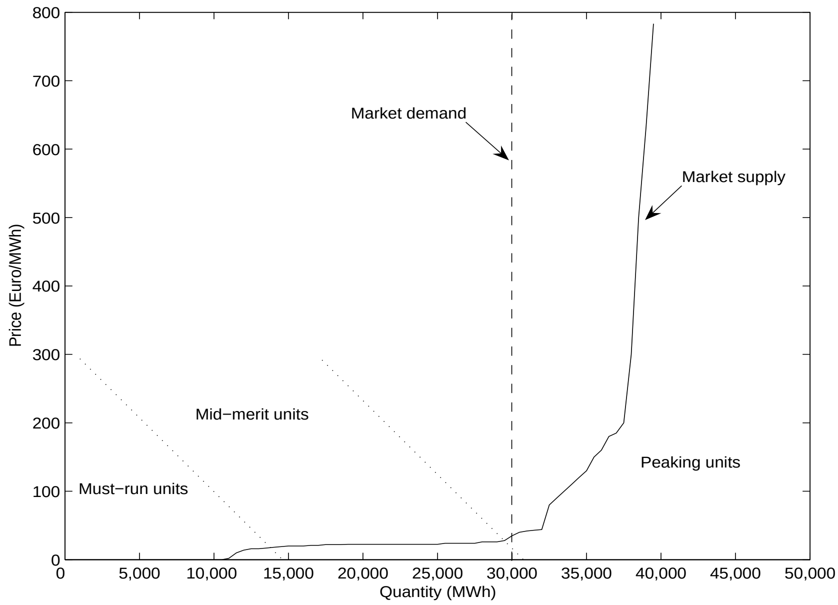

Figure 4.1: An example electricity supply stack

4.2.3 Characteristics of Electricity Prices

The nonstorability of electricity, along with the real-time matching mechanism of supply and demand, lead to several extraordinary features in electricity prices. In this subsection, we will discuss some of these features, namely, the high volatility, mean reversion, seasonalities, jumps and local prices.

High and Varying Volatility

Volatility is a measure of price fluctuation. Defined as the standard deviation of log returns, volatility measures the magnitudes of percentage changes in prices over time. As a unitless measure, it allows the comparison of relative price movements in different markets.

Prices in the energy market are marked by a volatility that is both high and variable over time. This property remains particularly pertinent in the electricity market. It is not unusual to notice annualized volatility of more than 1,000% in hourly spot prices. On the APX market for example, in early January 2000, the sudden jump from average levels of 44 Euro/MWh to 474 Euro/MWh translated to volatilities in the order of 1,500%. Volatilities of hourly prices in some markets can reach 3,000%.

As we discussed in Subsection 4.2.1 and 4.2.2, nonstorability of electricity requires electricity supply and demand be matched on a real time basis, subject to transmission capacity constraints. Since electricity demand varies frequently to usage pattern and weather conditions, power plant outage may happen unexpectedly and transmission capacity is limited, electricity prices vary wildly accordingly.

Another feature of the price volatility is the dependence of volatility on price. It is observed that price fluctuations are accompanied by fluctuations in price volatility. We call this phenomenon heteroscedasticity. In peak hours when supply is relatively tight and prices are high, power plant outages and transmission congestion are most likely to occur. Furthermore, it will need some time to restart the plant or transmission line and thus bring supply and demand back into balance. With this reasons, extreme prices are likely to be followed by other extreme prices. This phenomenon is also known as the clustering effect[2].

Mean Reversion

However volatile the electricity prices are, they tend to fluctuate around the average level. This feature is called mean reversion. Most commodities' prices have the feature of mean reversion [Sch97] [Pin99]. Mean reversion in electricity prices have been agreed by many researchers (see for example in [JB99], [EPV02], [WBT04], [HM04] and [GR06]).

Changes in electricity prices are directly related to the marginal cost of production. Marginal cost and average full cost per unit of production are considered to drive the long-term electricity price trend. Large deviations either upwards or downwards from the mean value are gradually mitigated through the influence of fundamental market forces. As demand increases, supply reacts by launching the generation units with respect to their marginal cost. The less efficient generation units, i.e., units with the highest marginal cost, will be started only in case of necessity. In short time, price fluctuations are corrected by demand and supply balancing.

[2] [KR05] declares an inverse leverage effect which describes the asymmetric response of volatility to positive and negative shocks. Positive shocks amplify the conditional variance of the process more than negative shocks.

Seasonality

Electricity prices exhibit pronounced periodic patterns on time scales from a hour to several months. Nonstorability of electricity excludes the possibility of a deferred use. Hence, electricity prices fluctuate cyclically in response to the variation in demand, which is in turn influenced by the weather and other exogenous factors with various cyclical fluctuations. In markets where hydro supply plays a significant role, the cyclical rainfall and thus level of reservoirs also contribute to the seasonal effect.

Seasonal patterns in electricity prices are categorized into three time scales: intra-day, day of a week and annual seasonalities. The intra-day pattern comes from the strong change in consumption during day and night. The on-peak and off-peak hours definition differs with markets. In the German spot market EEX, the on-peak hours range from 8 a.m. to 8 p.m., and the off-peak hours from 8 p.m. to 8 a.m. of the next day. In the Dutch spot market APX, the on-peak hours range from 7 a.m. to 11 p.m., and off-peak hours from 11 p.m. to 7 a.m. of the next day. The day of a week pattern depends on differences in industrial activities between weekdays and weekends. Finally, the annual seasonality is related to weather conditions and electricity consumption patterns. In some markets, two peaks can be observed in winter and summer time during one year. While in some other markets, only one peak period is observed in winter.

Jumps

Price jumps are occasionally observed in electricity spot markets. Given the feature of nonstorability, electricity prices are much more driven by spot demand and supply considerations than any other commodities. The demand of electricity in the short term is very inelastic. Therefore, prices can rise very quickly to an extremely high level when the balance is disturbed from time to time by disruption in transmission, generation outages, extreme weather, or a combination of these circumstances. When the relevant asset is returned to service, or demand recedes, prices quickly return to a typical level. Price jumps are then observed. In some sense, jumps in electricity prices can be considered the joint result of high price volatility and quick mean reversion.

Market-Specific Prices

The electricity markets in the world vary from one place to another. On the demand side, different environmental conditions and life-styles lead to different consumption patterns. On the supply side, different fuel sources, technologies, transmission networks, and ownership structures make regional supply curves different. Additionally, regulations vary from market to market. Market structures and rules are important drivers of the behavior of prices in a competitive electricity market [Wol97]. These factors make the price behavior very specific to a certain market. In another word, although electricity prices in different markets share some common features, their behaviors may differ remarkably. For example, electricity prices in Nordic countries are often found to be significantly different from continental European countries.

The fact that prices vary from market to market suggests us to accentuate the economical fundamentals of individual electricity market when examining its price behaviors. Models that perform well in one market do not necessarily perform well in other markets.

4.3 Electricity Price Modeling

In this section, we survey different models for electricity spot prices. In the beginning, we compare different approaches to electricity price modeling. Then we discuss on how to deal with seasonality in electricity prices. After that, we introduce various spot price models and discuss on how they can capture electricity price properties.

4.3.1 Electricity Modeling Approaches

The modeling of electricity prices can be generally classified into three different approaches: (1), reduced-form models, which attempt to model the price time series directly; (2), equilibrium models, which use the system fundamentals to determine marginal costs and translate these to prices; and (3), hybrid models which attempt to combine the reduced-form and fundamental models.

Fundamental models were developed under the regulated power market system and are generally optimization procedures. This approach is to minimize the total cost of production, subject to the power demand for the region as well as operational and environmental constraints. The electricity prices are determined by the marginal cost of production.

Fundamental models are typically very detailed and demanding in data, including loads, generation profile and environmental, operational and transmission constraints. The model must take into account the uncertainties around each of these driving factors as well. Different scenarios require separate model specification. This enormous computational expense renders simulation approaches very costly in time. In addition, there is virtually no mechanism for inclusion of existing market data in these simulations. An example of this approach can be seen in [BL02].

The class of reduced-form models is generally adapted from traditional financial markets. This approach attempts to specify the spot price process directly from historical data. These models generally attempt to fit power prices into the framework of financial models, mainly the interest rate term structure models, which have been well studied and are often tractable. The primary building block in this framework is a Brownian motion, or a Geometric Brownian motion.

Reduced-form models are based purely on the market data, thus they need to be modified to account for changes in underlying system conditions. Due to the easy data availability, these models are the most popular models for both researchers and practitioners. Due to data availability constraint, we restrict our modeling to the reduced-form approach in this thesis.

The hybrid models are the combination of the reduced-form and fundamental models. The combination of market fundamentals and historical price data gives an explicit link to the market. The hybrid model is shown by some researchers to be a promising approach for electricity price modeling [And02].

4.3.2 Treatment of Seasonalities

In reduced-form models, the starting point is to seek stochastic processes that can capture the characteristics of electricity prices. Seasonality represents a deterministic feature of the price, and thus can obscure the underlying price processes. As long as seasonality is observed, it has to be described and eliminated. When the calibrated model is used for price forecast, the final predicted price is the forecasted stochastic price plus the seasonal components.

Seasonal Effects Detection

The spot price at time t, represented by X_t, can be decomposed into two parts [Pil98] [LS02]. The first part is the stochastic process P_t, referring to the spot price in case where no seasonal effect exists. We will discuss the form of P_t in next subsection. The second part is the predicable, cyclical component, which can be represented by a deterministic function of time, $F = f(t)$. The observed price is then described by[3]

$$X_t = P_t + f(t) \tag{4.1}$$

When the seasonal effects are stripped from the observed prices, the seasonalities can be modeled separately from the underlying "pure" stochastic process. Thus, the first step is to detect and identify the seasonal properties in the price series. With increasing level of complexity, three methods can be employed to detect the frequency of seasonality [BD01], namely, the visual inspection, the dummy variable methodology and the Fast Fourier transform.

The visual inspection method. With the visual inspection method, the first step is to draw the price curve and to visually observe its behavior. We can point out regularities in the frequency of appearance of the highest and lowest values. To verify seasonality, a period of time that seems relevant to define a cycle is determined and the price curve of each of this period is superimposed. With this simple method, we observe if there are intra-day, day of a week, and annual seasonalities.

The dummy variable method. The dummy variable method relies on regressions. In detecting the annual seasonalities, for example, we assume that prices are on average the same through the whole year except in December. We then need to do a regression with the following equation

$$X_t = \alpha_t + \beta_1 D_t + e_t \tag{4.2}$$

where X_t is the price at time t, $D_t = 1$ if P_t is the price during December and 0 otherwise, e_t is the regression residual and $e_t \backsim N(0, \sigma^2)$. The average price during the special month is

$$E[P_t \mid D_t = 1] = \alpha_t + \beta_1 \tag{4.3}$$

[3]Seasonality can also be modeled as a multiplicatively separable component. In this case, the observed electricity price is described as

$$X_t = P(t)f(t).$$

and the average price during the rest of the year is

$$E[P_t \mid D_t = 0] = \alpha_t \tag{4.4}$$

If $E[P_t \mid D_t = 1]$ is significantly different from $E[P_t \mid D_t = 0]$, i.e., β_1 is significantly different from 0, then seasonality is identified in the specified month, December.

The Fast Fourier Transform. The Fourier transform introduces the idea that any stationary time series can be decomposed into a sum of sine and cosine terms. The continuous time Fourier transform is

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-i\omega t}dt \tag{4.5}$$

where $X(\omega)$ is the frequency domain (amplitude) signal, $x(t)$ is the initial data, i is the imaginary unit in complex algebra, and ω is the real frequency.

The Fast Fourier transform (FFT) is used to solve the numerical approximation of the continuous Fourier transform. In Matlab 7.0, the FFT routines can be used to obtain the frequency spectrum of the price series, with the high amplitudes in the spectrum indicating possible seasonality periods. The FFT method is complex but general and can be applied to any time series.

Seasonality Modeling

Next, we introduce two methods, namely, the dummy variable method and the sinusoidal function method, for seasonality modeling.

Dummy Variable Method. Knittel and Roberts [KR05] describe seasonal effects in electricity prices by using a series of dummy variables. They use a mean reverting model for the electricity price P_t, which we will discuss in the next subsection. These dummy variables allow the mean price level, $\mu(t)$, to vary across time.

$$\begin{aligned} \mu(t) \;=\; & \alpha_1 1(t \in Peak) + \alpha_2 1(t \in OffPeak) + \alpha_3 1(t \in Weekend) \\ & + \alpha_4 1(t \in Fall) + \alpha_5 1(t \in Winter) + \alpha_6 1(t \in Spring) \end{aligned} \tag{4.6}$$

where $1(.)$ is the indicator function. For example, $1(t \in Peak) = 1$, if t is within peak hours, and $1(t \in Peak) = 0$, otherwise.

For the parameter estimation of the mean reverting model, we need to discretize the mean reverting process into an AR(1) model [DP94]. We can incorporate equation (4.6) into the regression, viewing α_t as a variable consisting of six binary variables. The statistical significance of $\alpha_i (i = 1, 2, 3, 4, 5, 6)$ indicates the degree of seasonality with respect to different time horizons.

The Sinusoidal Function Method. Pilipovic [Pil98] proposes a method to capture the seasonal behavior in energy prices by using sinusoidal functions. In the case of electricity, this method has to be adapted to various seasonal patterns. Pilipovic gives a description of $f(t)$ in equation (4.1) as

$$f(t) = \beta_A(\cos(2\pi\theta(t - t_A))) + \beta_{SA}(\cos(4\pi\theta(t - t_{SA}))) \tag{4.7}$$

where β_A is the annual seasonality parameter, t_A is the annual seasonality centering parameter (time of annual peak), β_{SA} is the semi-annual seasonality parameter, and t_{SA} is the semi-annual seasonality centering parameter (time of semi-annual peak).

The Pilipovic method is capable of modeling annual periodicity effects. The intra-day and day of a week effects can also be incorporated by adding corresponding dummy variables to the right-hand side of equation (4.7) [LS02].

4.3.3 One-Factor Models

One-factor models are the simple models which can model the randomness in electricity prices. Some one-factor models can also capture the mean reversion in electricity prices. We will introduce the Arithmetic Brownian Motion model, the Geometric Brownian Motion model, the Ornstein-Uhlenbeck model, the Geometric Ornstein-Uhlenbeck model, and the Mean Reverting Proportional Volatility model.

Arithmetic Brownian Motion

The Arithmetic Brownian Motion (ABM) model assumes that the electricity spot price follows a Brownian motion process, or a continuous-time random walk. The fundamental differential equation is given by

$$dP_t = \mu dt + \sigma dZ_t \tag{4.8}$$

where dP_t is the change in the spot price from time t to time $t + dt$, μ is the instantaneous drift term, σ is the standard deviation of electricity prices, dZ_t is the increment of a standard Wiener process, so $dZ_t \sim N(0, \sqrt{dt})$.

The ABM model is the first formal mathematical model of financial asset prices. Thus it is the foundation of most other models.

Geometric Brown Motion

The Geometric Brownian Motion (GBM) model is the most used model in the financial markets. The differential equation for this model is

$$dP_t = \mu P_t dt + \sigma P_t dZ_t \tag{4.9}$$

where dP_t is the change in the spot price from time t to time $t + dt$, μ is the instantaneous drift term, σ is the volatility of electricity prices, dZ_t is the increment of a standard Wiener process, so $dZ_t \sim N(0, \sqrt{dt})$.

According to equation above, the change in the price over time dt consists of two terms. The first term $\mu P_t dt$ is the drift, or the deterministic term. The second term $\sigma P_t dZ_t$ is the stochastic, or random term. Both the drift and stochastic terms are proportional to the spot price level at time t.

Let $x_t = \ln(P_t)$ and apply Ito's Lemma to x_t, we get

$$dx_t = (\mu - \frac{\sigma^2}{2})dt + \sigma dZ_t \tag{4.10}$$

Solving for x_t, we can derive the spot price at time t , P_t, contingent on the spot price at time 0, P_0.

$$P_t = P_0 \exp((\mu - \frac{\sigma^2}{2})t + \sigma W_t) \tag{4.11}$$

where W_t is normally distributed random variable with mean zero and variance t.

The logarithm of P_t is thus normally distributed, and P_t has a lognormal distribution.

The simplicity and tractability of a GBM model lead to its wide usage in asset pricing. For example, the Black-Scholes option pricing model is based on the assumption that the underlying asset price follows a GBM process.

The drawback of the Brownian motion and GBM models is that they cannot capture some of the important features of electricity spot prices such as mean reversion and jumps.

Ornstein-Uhlenbeck Model

The Ornstein-Uhlenbeck (O-U) model is specially suited for modeling the mean reverting behavior in prices. The O-U process is expressed by

$$dP_t = \kappa(\theta - P_t)dt + \sigma dZ_t \tag{4.12}$$

where dP_t is the change in price from time t to time $t + dt$, κ is the mean reversion rate, θ is the mean price, σ is the volatility of electricity prices, dZ_t is the increment of a standard Wiener process, so $dZ_t \sim N(0, \sqrt{dt})$.

The O-U model is consistent with the mean reversion in electricity prices over time. The larger the deviations from the mean price, the stronger the mean reversion effect will be. Moreover, the O-U model is analytically tractable. The price at a future time T, P_T, conditional on the initial price P_0, is an explicit solution to equation (4.12):

$$P_T = e^{-\kappa T}P_0 + (1 - e^{-\kappa T})\theta + \sigma e^{-\kappa T}\int_0^T e^{\kappa t}dZ_t \tag{4.13}$$

It is easy to see that P_T follows a conditional normal distribution. The conditional mean and variance are given by

$$E_0[P_T] = e^{-\kappa T}P_0 + (1 - e^{-\kappa T})\theta \tag{4.14}$$

$$Var_0[P_T)] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa T}) \tag{4.15}$$

Equation (4.13) can be interpreted as a weighted average of the current process value P_0, and the long-term mean, θ, with weights obtained from discounting at an adjusted speed rate, κ. The half-time at which the expected value $E_0[P_T]$ is halfway between the long-term mean, θ, and the current price, P_0, is given by

$$T_{half-life} = (\ln 2)/\kappa \tag{4.16}$$

According to equation (4.15), the variance of P_T is increasing in T and bounded from above by $\frac{\sigma^2}{2\kappa}$.

Geometric Ornstein-Uhlenbeck Model

The Geometric Ornstein-Uhlenbeck model simply assumes that the logarithm of the price follows the O-U process. The geometric O-U process is expressed by

$$d\ln(P_t) = \kappa(\theta - \ln(P_t))dt + \sigma dZ_t \tag{4.17}$$

where $d\ln(P_t)$ is the change in logarithm of price from time t to time $t+dt$, κ is the mean reversion rate, θ is the mean value of the logarithms of spot prices, σ is the volatility of the logarithms of spot prices, dZ_t is the increment of a standard Wiener process, so $dZ_t \sim N(0, \sqrt{dt})$.

Alike an O-U model, a Geometric O-U model is able to capture the mean reversion in electricity prices.

Inhomogeneous Geometric Brownian Motion Model

If we take the first term from the O-U model in equation (4.12), which gives the mean reversion property, and the second term from the GBM model in equation (4.9), which allows the model to satisfy the price-proportional volatility, we have an Inhomogeneous Geometric Brownian Motion (IGBM) model [BD02] [AC05]. The stochastic process for the IGBM model is given by:

$$dP_t = \kappa(\theta - P_t)dt + \sigma P_t dZ_t \tag{4.18}$$

where θ is the level electricity price tends to in the long run, κ is the mean reverting rate, σ is the instantaneous volatility of electricity price, dZ_t is the increment of a standard Wiener process.

The IGBM model is capable of capturing the mean reversion and price-proportional characteristics in electricity prices. It differs from the O-U process by the term $\sigma P_t dZ_t$ in the differential equation. This specification precludes the possibility of negative prices [AC05]. In a special case when $\theta = 0$ and $\alpha = -k$, the inhomogeneous GBM process reduces to a standard GBM process.

4.3.4 Two-Factor Models

In order to model more features of electricity prices, people use two-factor models. Some two-factor models allow the volatility or the long-term price to be stochastic. Other two-factor models describe the short-term and long-term variations in electricity prices.

Stochastic Volatility

A one-factor GBM or O-U model can be extended to a two-factor model by introducing a second variable. For example, a stochastic volatility can be incorporated [DJS01]. In [DJS01], electricity prices follow an O-U process, capturing the mean reversion property, while the volatility is described by another O-U process. Thus, we have

$$dP_t = \kappa_P(\theta - P_t)dt + V_t dZ_t \quad (4.19)$$

$$dV_t = \kappa_v(m - V_t)dt + \sigma dW_t \quad (4.20)$$

where dP_t is the change in price from t to $t + dt$, κ_P is the mean reversion rate of electricity price, θ is the long-term mean of electricity prices, κ_P is the mean reversion rate of the electricity prices, V_t is the volatility of electricity prices, κ_v is the mean reversion rate of the volatility, m is the long-term mean of volatility, σ is the volatility of volatility, dZ_t and dW_t are increments of two independent standard Wiener processes.

Pilipovic Model

The two-factor model presented by Pilipovic [Pil98] allows the long-term mean level of electricity price to be stochastic. The first factor is the spot price, which is assumed to be mean reverting toward the long-term equilibrium price. The second factor is the long-term equilibrium price, which is lognormally distributed. The dynamics of the prices are given by

$$dP_t = \kappa(L_t - P_t)dt + \sigma P_t dZ_t \quad (4.21)$$

$$dL_t = \mu L_t dt + \xi L_t dW_t \quad (4.22)$$

where dP_t is the change in price from t to $t + dt$, κ is the mean reversion rate of electricity prices, L_t is the long-term mean electricity price at time t, σ is the volatility of electricity prices, μ is the drift of the long-term equilibrium price, ξ is the volatility of L_t, dZ_t and dW_t are increments of two independent standard Wiener processes.

Conditional on the spot price and the long-term equilibrium price observed at time t, we can obtain the expected spot price at a future time T, i.e.,

$$E_t[P_T] = P_t e^{-\kappa(T-t)} + \frac{\kappa}{\kappa + \mu} L_t(e^{u(T-t)} - e^{-\kappa(T-t)}) \quad (4.23)$$

If we let the long-term equilibrium price volatility, ξ , be zero, the Pilipovic two-factor is then reduced to the one-factor IGBM model.

Schwartz-Smith Model

The Schwartz and Smith [SS00] two-factor model assumes the variation in commodity prices are jointly explained two components: the short-term deviation and the long-term uncertainty. Let P_t denote the spot price of a commodity at time t, the spot price is thus decomposed into two stochastic factors as

$$\ln(P_t) = \chi_t + \xi_t \tag{4.24}$$

where χ_t is the short-term deviation in prices and ξ_t is the long-term equilibrium price.

The short-run deviation χ_t is assumed to revert toward zero following an O-U process

$$d\chi_t = \kappa\chi_t dt + \sigma_\chi dZ_\chi, \tag{4.25}$$

and the equilibrium price ξ_t is assumed to follow an ABM process

$$d\xi_t = \mu_t dt + \sigma_\xi dZ_\xi \tag{4.26}$$

where dZ_χ and dZ_ξ are increments of two correlated standard Wiener processes with $dZ_\chi dZ_\xi = \rho_{\chi\xi}dt$, σ_χ and σ_ξ are the volatility of the short- and long-term prices, respectively, and μ_t is the growth rate of the equilibrium price. The mean reversion coefficient κ describes the rate at which the short-term deviations are expected to disappear.

These two factors are not directly observable in the markets. Movements in prices for long-maturity futures contracts provide information about the equilibrium price level, and differences between the prices for the short- and long-term contracts provide information about short-term variations in prices. Thus the parameters may be estimated from spot and futures prices. Kalman filtering techniques are used to estimate these unobservable state variables [SS00] [MT02]. The short-term/long-term model is proved to be equivalent to the stochastic convenience yield model in [GS90], but the short-term/long-term model is easier to interpret and work with.

Furthermore, the short-term and long-term two-factor model includes the standard GBM and O-U models as special cases when there is uncertainty about only one of the two factors.

Jump Diffusion Models

The jump diffusion model was first suggested by Merton [Mer76] to price a stock option. The key assumption made in [Mer76] is that the jump component of the asset's return represents non-systematic risk. This risk can be diversified away and is not priced in the economy. The probability of occurrence of jumps during a time interval Δt is given by

$$\begin{aligned} Prob[\text{no events occur during } (t, t+\Delta t)] &= 1-\lambda_t + O(\Delta t) \\ Prob[\text{the event occurs once during } (t, t+\Delta t)] &= \lambda_t + O(\Delta t) \\ Prob[\text{the event occurs more than once during } (t, t+\Delta t)] &= O(\Delta t) \end{aligned}$$

where $O(\Delta t)$ is the asymptotic order symbol, which can be defined by $\Psi(\Delta t) = O(\Delta t)$ if $\lim_{h\to 0}[\Psi(\Delta t)/\Delta t] = 0$, and λ is the mean number of arrivals per unit time, i.e., the intensity of the process.

The jump diffusion model is built by adding a Poisson jump term to the mean reverting model. This specification implies that when an event occurs, there is an instantaneous jump in the price of random size assumed independent of the lognormal diffusion process. The jump diffusion model is given by

$$dP_t = \kappa(\theta - P_t)dt + \sigma dZ_t + J_t dq_t \tag{4.27}$$

where dq_t is a Poisson process with an intensity of λ and J_t is the jump size, which follows a lognormal distribution, i.e., $\ln(J_t) \sim N(\mu, \rho^2)$, μ and ρ^2 are the mean and variance of the jump size J_t.

The jump intensity λ can be assumed constant for simplicity [Bar99], and it can also be allowed to vary over time [KR05], reflecting the fact that jumps are more likely to occur at high demand times.

Due to their capability of capturing important features of electricity prices, mean reverting jump diffusion models are chosen by many modelers (see for example in [Bar99], [Den99], [CW00], [EPV02] and [WBT04]).

One criticism of jump diffusion models concerns the mingling of mean reversion components and jumps, which may lead to misspecification of the true mean reverting behavior. For example, a sharp price decrease may be considered both as a result of mean reversion force and the downward jump movements. In order to distinguish these two effects, a level-dependent signed-jump model is proposed in [GR06], but much more complexity is introduced. An alternative method to disentangle jumps and mean reversion is to use a regime switching model, which we will discuss next.

Regime Switching Method

A regime switching model allows electricity prices to jump discontinuously between different states, with state dependent probabilities. Classic regime switching models are studied by Hamilton [Ham94]. As a variant of Hamilton model, a two-state switching model is proposed in [EM98] to model electricity prices. The model is given by

$$x_t - \mu_{s(t)} = \phi(x_{t-1} - \mu_{s(t-1)}) + \varepsilon_t \tag{4.28}$$

where x_t is the natural logarithm of the daily spot price of electricity, $s_t = 1, 2$ is the indicator of states, ϕ is the autoregressive coefficient, $\varepsilon_t \sim N(0, \sigma^2_{s(t)})$, $\sigma^2_{s(t)}$ is the variance of $x_t - \mu_{s(t)}$, and $\mu_{s(t)}$ is the mean value of x_t in each state.

Rather than the isolated and independent jumps specified in jump diffusion models, the two-state switching model allows two states which can persist. This is important because jumps in electricity prices are often driven by extreme weather or plant outages, which tend to persist for a period of time. In order to model the dynamic probabilities of price switching between regimes, we need to specify a matrix of conditional jump probabilities. This matrix is given by

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix} \tag{4.29}$$

where $p_{ij}(i, j = 1, 2)$ is the probability of x_t being in state i conditional on x_{t-1} being in state j.

Extreme Value Theory

For a long time, extreme value theory (EVT) has been used in different areas to model the tails of a distribution that are induced by extreme events. Recently, applications of this theory have appeared in finance and insurance, where rare events may cause catastrophic losses.

The EVT method is not describing the entire distribution of the prices, but focuses only on modeling the tails. As such, the entire price series is modeled as a combination of ordinary time series and an EVT based tail model. For example, the peak-over-threshold (POT) method in EVT deals with those events in a certain data set that exceed a high threshold and model these separately from the rest of the observations. After fitting a time series model to price return data, the residual series, which is close to being independently identically distributed, is then modeled by the EVT method.

The EVT method is found to be able to model the extreme behaviors in electricity prices [CG05] [Han05].

4.4 Empirical Tests of Two Models

In this section, we empirically examine the electricity price behavior in the German and Dutch markets and compare the two most popular models for electricity spot prices: the mean reverting jump diffusion model (MRJD) and the two-regime switching model (RS). These two models are capable of capturing the volatility, mean reverting, and jumps, and are mathematically tractable. We use in-sample goodness-of-fit and out-of-sample forecasting error as the criteria to compare the performance of the two models.

4.4.1 Market Data and Price Properties

The main data to be used in this empirical study are the daily average prices from the Dutch spot market APX (Amsterdam Power Exchange) and the German spot market EEX (European Electricity Exchange). For the study of the intra-day price patterns, we also use the hourly prices on some days.

The whole sample consists of data from January 1, 2001 to December 31, 2004. The reason why we chose this sample is twofold. On the one hand, deregulation was in its infancy in earlier years before 2001, and the spot markets at these two exchanges were not liquid. On the other hand, the carbon dioxide emission trading was introduced in European Union in 2005, which is conceived as a structural change in the markets. From then on, market participants have to face CO_2 price movements and further regulative uncertainty on CO_2[4]. To take this fundamental change into account, new factors must be included in the pricing models. This is beyond the scope of this thesis.

The curves of the daily average prices in the sample period are plotted in Figure 4.2. From visual inspection, electricity prices appear highly volatile with occasional jumps and mean reversion. The statistical properties of

[4]During the first stage from 2005 to 2007, the emission allowances were granted to energy producers for free. Affluent allowances at the first stage have led to intensive debate on the allocation plan for the second stage starting 2008.

logarithm of the daily average prices are reported in Table 4.1. The high standard deviations of electricity prices imply high volatilities. For example, a standard deviation of 0.50 in the logarithm of daily price in APX translates into an annual volatility of $0.50 \times \sqrt{365} = 947\%$. This is far more volatile than other commodities.

Table 4.1 also reports the non-normality of the electricity prices and returns. The skewness is significantly different from zero. The kurtosis is much greater than the level of a normal distribution, 3. The Jarque-Bera statistic[5] also rejects a normal distribution. We also performed the Q-Q plot test, which compares the real distribution of the sample with a normal distribution. The Q-Q plots are shown in Figure 4.3. The fat tails in real distributions deviate from normal distributions, which are represented by the dotted straight lines.

Next, we study the cyclical patterns in electricity prices. The intra-day hourly price patterns are illustrated in Figure 4.4, with a sample of one-week hourly prices of APX (from December 16, 2002 to December 22, 2002). During each day, prices climb from about 8:00 am, to their first peak at about 11:00 am. After the second peak period about 6:00 p.m., the prices decrease until the next morning.

Figure 4.4 also shows that prices on weekdays are higher than on weekends. The day of a week price patterns can also be inspected by plotting the autocorrelations of returns in daily average prices[6]. As shown in Figure 4.5, the autocorrelations of price returns do not fade away. This is quite different from most financial products. The salient 7-day cycle in autocorrelations suggests the day of a week price patterns.

We show the annual seasonalities in Figure 4.6 by plotting the monthly average prices in the sample period. Generally, prices are higher in peak months, especially in winter and sometimes in summer, and are lower in shoulder months.

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[5] Jarque-Bera Statistic is calculated as $\frac{N}{6}(S^2 + \frac{(K-3)^2}{4})$, where S is the skewness, K is the kurtosis and N is the number of observations.

[6] The autocorrelation function of the price return series is given by

$$acf(r,k) = \frac{\sum_{t=k+1}^{N}(r_t - \bar{r})}{\sum_{t=1}^{N}(r_t - \bar{r})^2}$$

where r_t is the electricity price return, k is the number of time lags, N is the number of observations and $\bar{r}$ is the average of r_t.

From Figure 4.2, we can observe by visual inspection that electricity prices tend to revert to a mean level. In order to examine mean reversion in electricity prices, we perform an Augmented Dick-Fuller (ADF) test with 5 time lags with the price data. The ADF(5) test results reported in Table 4.2 rejects a unit root, which implies a random walk. In other word, the autoregressive coefficient in an AR(1) model will be significantly different from zero and mean reversion applies.

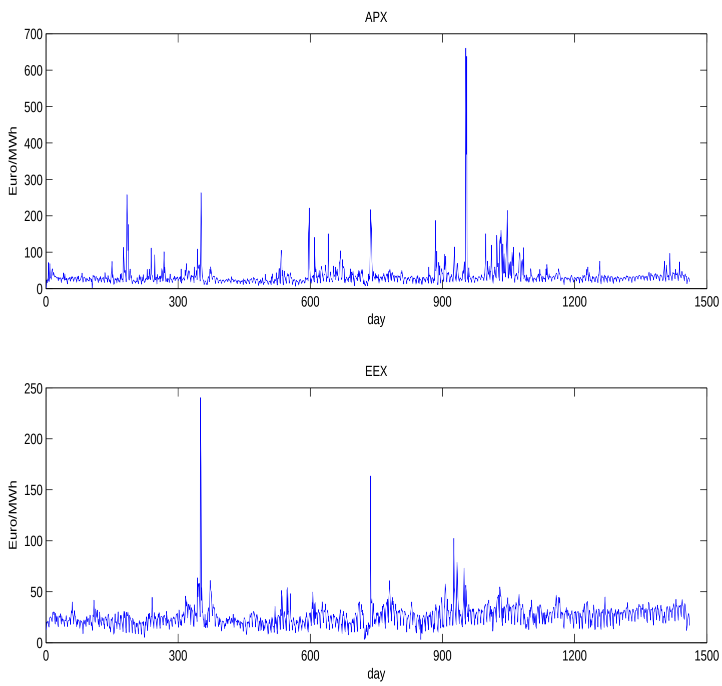

Figure 4.2: Daily average prices in APX and EEX

	APX		EEX	
	$ln(P_t)$	$ln(P_t/P_{t-1})$	$ln(P_t)$	$ln(P_t/P_{t-1})$
Mean	3.40	0.00	3.19	0.00
Median	3.37	-0.03	3.23	-0.04
Minimum	0.72	-2.53	1.14	-1.96
Maximum	6.49	3.54	5.48	2.37
Standard deviation	0.50	0.46	0.38	0.35
Skewness	1.08	0.78	-0.32	0.86
Kurtosis	5.17	6.09	3.49	4.38
Jarque-Bera *	571.05	730.07	40.31	295.42

*The Jarque-Bera Statistic of a normal distribution is zero. The Jarque-Bera Statistic follows a Chi-squared distribution with 2 degrees freedom. At 5% confidence level, the critical value to reject the null hypothesis of a normal distribution is 5.991.

Table 4.1: Statistics of electricity daily average prices

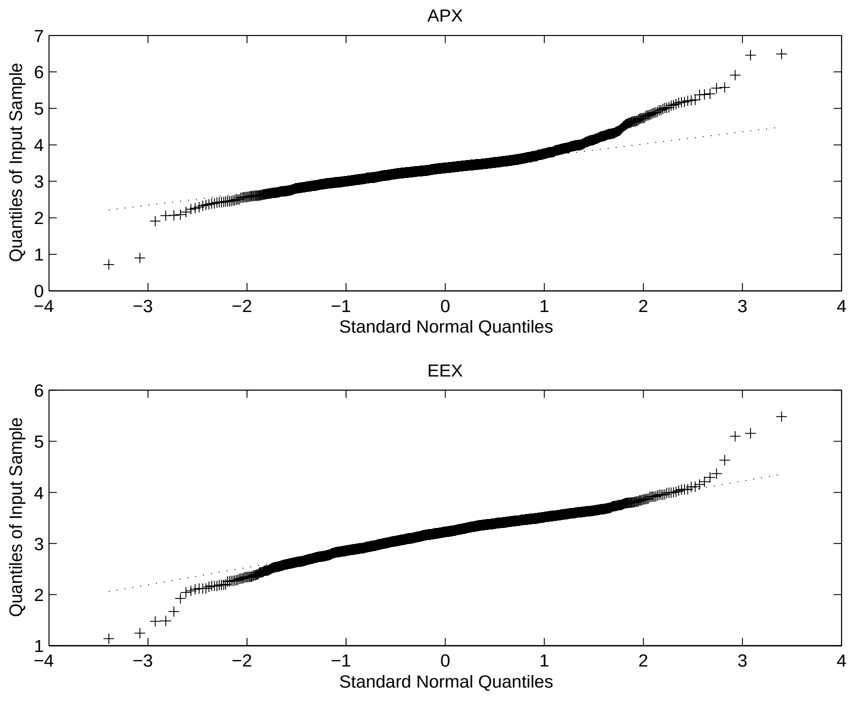

Figure 4.3: Q-Q plots of the logarithm of electricity daily average prices

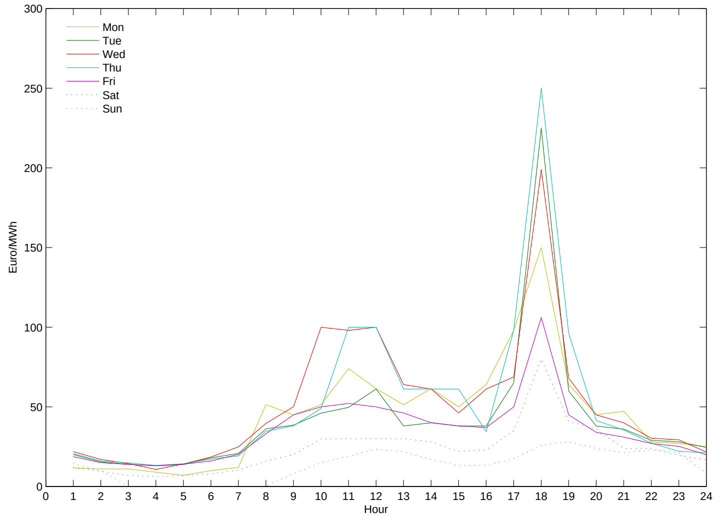

Figure 4.4: One-week hourly prices in APX (12/16/02-12/22/02)

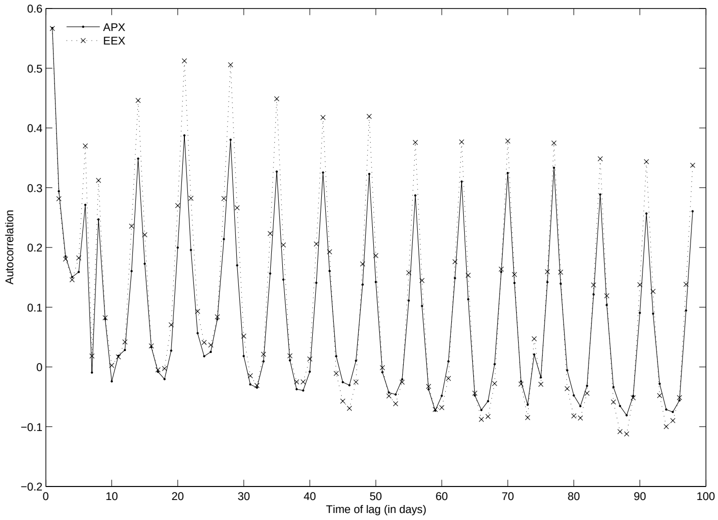

Figure 4.5: Lagged autocorrelation for price returns

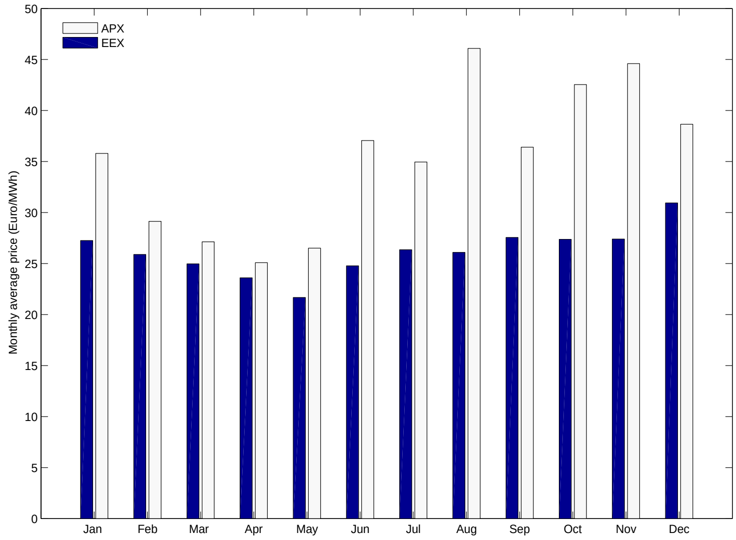

Figure 4.6: Monthly average prices

	ADF(5) test statistic	ADF(5) test critical values*	
APX	-5.5863	1%	-3.4382
EEX	-7.4235	5%	-2.8642
		10%	-2.5682

*MacKinnon critical values for rejection of hypothesis of a unit root. The null hypothesis of a unit root is rejected in favour of the stationary alternative, if the test statistic is more negative than the critical value.

Table 4.2: ADF(5) test of electricity daily average prices

In another test for mean reversion, we follow the method used by [Pin99], plotting the variance ratios against increasing time of lags, k. The ratio is calculated as

$$R_t = \frac{1}{k}\frac{Var(P_{t+k} - P_t)}{Var(P_{t+1-}P_t)} \tag{4.30}$$

If price follows a random walk, i.e., is not stationary, the variance of k-period differences should grow linearly with k and should approach 1 as k

increases. On the other hand, if price follows a stationary (mean reverting) process, the variance of k-period differences will approach an upper limit as k grows. So this ratio will fall to zero as k increases. The variance ratios of the daily average price in APX and EEX in Figure 4.7, decrease with increasing k toward zero. Hence, prices are mean reverting.

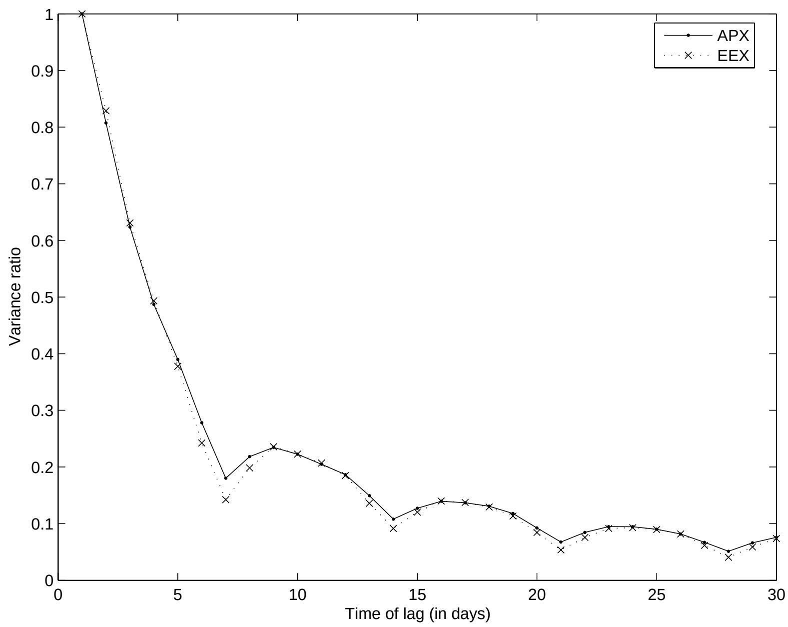

Figure 4.7: Variance ratios of electricity prices

4.4.2 Model Specification

In order to model seasonalities, we follow the spirit in [LS02] to decompose the logarithm of the observed spot price P_t into two parts: the stochastic component, X_t, and the seasonal deterministic component, S_t.

$$\ln P_t = X_t + S_t \tag{4.31}$$

Further, we decompose S_t into an annual pattern F_t and a weekly pattern W_t.

$$S_t = F_t + W_t \tag{4.32}$$

where

$$F_t = A_0 t + A_1 \sin(\omega_1 t + \phi_1) + A_2 \sin(\omega_2 t + \phi_2) \tag{4.33}$$

and $\omega_1 = \frac{2\pi}{365}, \omega_2 = \frac{4\pi}{365}$, $A_0, A_1, A_2, \phi_1, \phi_2$ are constant parameters, and A_0 represents the time trend, $A_1 \sin(\omega_1 t + \phi_1)$ and $A_2 \sin(\omega_2 t + \phi_2)$ represent the annual and semi-annual seasonal factors, respectively.

The weekly seasonality W_t is given by

$$W_t = \sum_{k=1}^{7} \gamma_{t,k} DP_k \tag{4.34}$$

where $\gamma_{t,k}$ are the dummy variables which take value 1 when $t = k$, and 0 otherwise. DP_k is the average deviation of the price of a particular day in a week from the weekly average prices.

We estimate the parameters with the data ranging from January 1, 2002 to November 30, 2004. The last month of 2004 is left for an out-of-sample test. The estimated seasonality parameters are reported in Table 4.3. As expected, the coefficients for weekend days in both markets are lower than coefficients for weekdays. It may seem strange that all weekly and annual parameters are negative. This is due to the simultaneous estimation of all seasonality parameters. The negative values are reasonable in order to correct the overshooting of the trend parameter A_0. This is affirmed by the good fitness in the curves in APX as shown in Figure 4.8. Results from EEX data are much similar, although they are not reported here.

From equation (4.31) and (4.32), we have

$$X_t = \ln(P_t) - S_t = \ln(P_t) - F_t - W_t \tag{4.35}$$

Substitute the estimated seasonality parameters in Table 4.3 into equation (4.33)-(4.35), we obtain the deseasonalized logarithms of the daily average prices X_t. This series is to be used to calibrate a stochastic process model. The curve in the bottom panel of Figure 4.8 gives the trajectory of X_t.

	APX	EEX
A_0	1.26E+00	2.52E+00
A_1	-0.1496	-0.0661
ϕ_1	-0.3068	-0.8256
A_2	-0.0417	0.0112
ϕ_1	-0.2960	-0.0824
DP_1	-0.1376	-0.4314
DP_2	-0.1370	-0.4405
DP_3	-0.1241	-0.4351
DP_4	-0.1373	-0.4336
DP_5	-0.1617	-0.4280
DP_6	-0.1897	-0.4596
DP_7	-0.1983	-0.4627
R^2	0.6629	0.7743
Standard Error	0.2491	0.1543
F value	52.9854	414.2670

Table 4.3: Estimated seasonality parameters

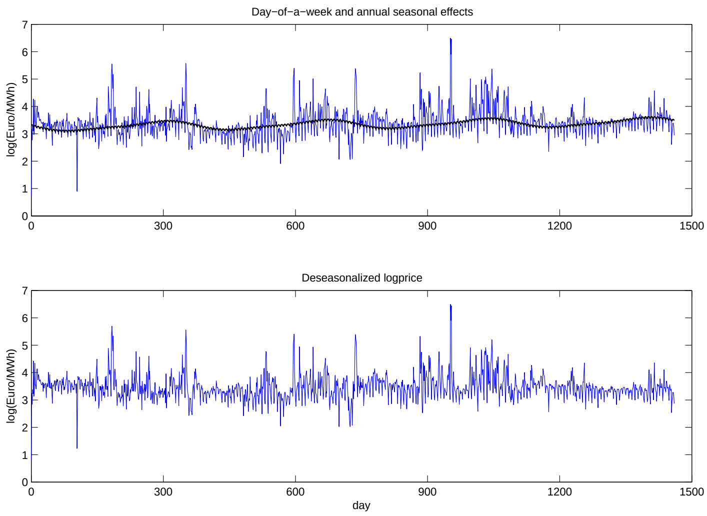

Figure 4.8: Deseasonalizing the log prices: APX

The first model we choose to calibrate is the MRJD model which is specified in equation (4.27). Following [CS00], we use the 3-σ method to identify jumps. That is, if the deviation of one price from the unconditional mean is greater than 3 times of the unconditional standard deviation, then this price is identified as a jump. The filtered jumps series are used to estimate the Poisson process and the remaining series are used to estimate the mean reverting diffusion process. The MLE method is used for parameter estimations.

The estimated parameters with data ranging from January 1, 2002 to November 30, 2004 are reported in Table 4.4. Figure 4.9 shows how the jumps are filtered out from the APX log prices. The middle panel of Figure 4.9 plots the jump-filtered remaining process. The bottom panel of Figure 4.9 gives the filtered jump series. Results from EEX data are much similar, except that less jumps are found there.

The second model we choose to calibrate is the two-state regime switching model. We define three types of regime switching models as variants to the classical Hamilton models. The first type is a mixture distribution model, in which we assume the log prices in both regimes follow a normal distribution. The second type is called De Jong-Huisman model [DH02], which assumes the log prices follow a mean reverting process in the normal regime and a normal distribution in the spike regime. The third type is the Ethier-Mount model [EM98], in which log prices in both regimes follow a mean reverting process.

Let X_{t,R_t} denote the underlying log price at time t. Define the regime index $R_t = 1, 2$. When $R_t = 1$, the price is in the normal regime. When $R_t = 2$, the price is in the spike regime. The two-regime switching models are mathematically described as

$$X_{t,R_t} = X_{t,1} \text{ or } X_{t,2} \tag{4.36}$$

where the transition matrix governing the switching between regime 1 and 2 is given by equqtion (4.29).

The processes for the three types of regime switching models are given by

	APX	EEX
k	0.6615 (0.0227)	0.6609 (0.0198)
m	3.4554 (0.0250)	3.4540 (0.0095)
σ	0.5606 (0.0079)	0.4478 (0.0058)
λ	0.0203	0.0091
μ	1.1314	-0.2985
ρ^2	0.6818	0.3388

Table 4.4: Estimated parameters for MRJD model (number in the parenthesis is the standard errors of the estimation)

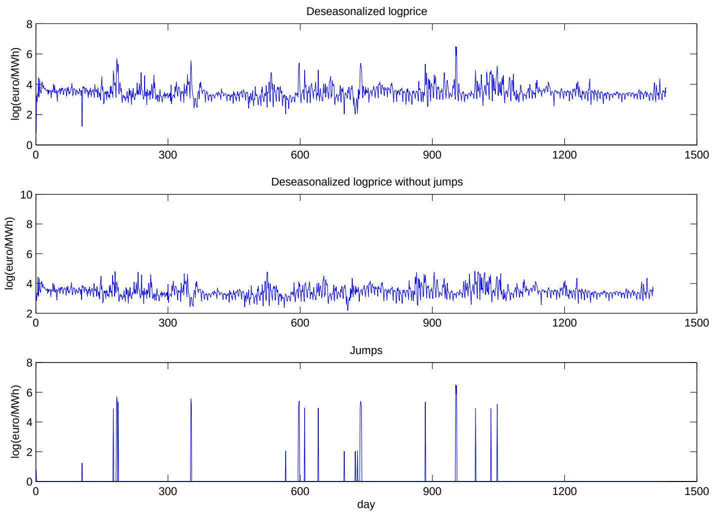

Figure 4.9: Jump filtering: APX

Mixture distribution model:

$$\begin{aligned} X_1 &= m_1 + \epsilon_1 \\ X_2 &= m_2 + \epsilon_2 \end{aligned} \tag{4.37}$$

where $\epsilon_i \sim N(0, \sigma_i^2), i = 1, 2$, m_1 and m_2 are the mean log price levels in the normal and spike regimes, respectively, and $m_1 < m_2$.

De Jong-Huisman model:

$$\begin{aligned} dX_{t,1} &= k_1(m_1 - X_{t,1})dt + \sigma_1 dZ_{t,1} \\ X_2 &= m_2 + \epsilon_2 \end{aligned} \tag{4.38}$$

where m_1 is the mean log price level in the normal regime, σ_1 is the volatility of log prices, $Z_{t,1}$ is a standard Brownian motion, m_2 is the mean of log price level in the spike regime, $\epsilon_2 \sim N(0, \sigma_2^2)$, and $m_1 < m_2$.

Ethier-Mount model:

$$\begin{aligned} dX_{t,1} &= k_1(m_1 - X_{t,1})dt + \sigma_1 dZ_{t,1} \\ dX_{t,2} &= k_2(m_2 - X_{t,2})dt + \sigma_2 dZ_{t,2} \end{aligned} \tag{4.39}$$

where m_1 and m_2 are the mean log price levels in the normal and spike regimes, respectively, k_1 and k_2 are the mean mean reversion rates in the normal and spike regimes, respectively, σ_1 and σ_2 are volatility of log prices in the normal and spike regimes, respectively, $Z_{t,1}$ and $Z_{t,1}$ are two independent Brownian motions, and $m_1 < m_2$.

We estimate the regime switching models by an Expectation Maximization (EM) algorithm. The EM method uses an iterative procedure that consists of two steps. In the first step, we assume an initial set of parameter values, $\hat{\theta}^0$, and calculate the posterior estimates of the probability $P(R_t = i | X_1, X_2, .., X_T, \hat{\theta}^0)$ that the process is in regime i at time t with knowledge of the complete data set $X_1, X_2, .., X_T$. The expected value of the log-likelihood can thus be calculated via the likelihood function. In the second step, we maximize the expected likelihood to obtain a new set of model parameters, $\hat{\theta}^1$. With the new vector $\hat{\theta}^1$, we start the next round of the algorithm to recalculate the expected value of the the log-likelihood, and so on. This EM algorithm iteratively improves an initial estimate $\hat{\theta}^n$ by constructing new estimates $\hat{\theta}^{n+1}$. When $\hat{\theta}^n$ converges, the maximum likelihood estimates of the parameters are obtained. Details of the EM estimation can be found in [DLR77] and [Ham94].

In order to avoid a local maximum of the likelihood function, we used several different random initial estimates, $\hat{\theta}^0$. The estimates are stable with the different initial settings. The EM estimated parameters with data from January 1, 2002 to November 30, 2004 are reported in Table 4.5.

APX			
	Mixture distribution	De Jong-Huisman	Ethier-Mount
p_{11}	0.9730	0.9849	0.9803
p_{22}	0.6311	0.4785	0.3683
k_1		0.7298	0.4913
m_1	3.4045	3.4162	3.4312
σ_1	0.3215	0.2987	0.4375
k_2			1.8442
m_2	4.5185	4.9547	4.9644
σ_2	0.3489	0.3210	0.2995
EEX			
	Mixture distribution	De Jong-Huisman	Ethier-Mount
p_{11}	0.9652	0.9634	0.9604
p_{22}	0.7534	0.6732	0.4206
k_1		0.8452	1.013
m_1	3.1015	3.2354	3.2746
σ_1	0.2684	0.2580	0.3157
k_2			1.2584
m_2	3.9818	4.1546	4.2432
σ_2	0.2566	0.2498	0.2012

Table 4.5: Estimated parameters for regime switching models

4.4.3 Performance Comparison

To compare the in-sample goodness-of-fit of the MRJD and regime switching models, we use a simulation-based approach. Based on the model calibration results in the previous subsection, we simulate 5000 price paths for the sample period with each of the models. We then compare the trajectories and statistics of the simulated prices with the real historical prices. By visual inspection, the simulated price paths from both MRJD and regime switching models resemble the real price curve quite well. The statistics of the simulated prices for APX data are reported in Table 4.6, together with historical data. We find that the statistics from the simulations of regime switching models are generally closer to historical statistics than those of MRJD model. Especially, the skewness and kurtosis in log prices are better captured by regime switching models. Finally, the De Jong-Huisman model outperforms all other models. These findings are supported by simulations for EEX market as well, although the EEX results are not reported here.

	Historic data	MRJD	Mixture distribution	De Jong-Huisman	Ethier-Mount
Jumps	NA	33	135	52	43
Mean	3.4031	3.4712	3.4550	3.4624	3.4688
Variance	0.2489	0.4335	0.2069	0.2313	0.2211
Max	6.4928	6.3441	5.2677	5.6003	5.4562
Min	0.7178	1.1352	2.1238	2.0832	2.0132
Skewness	1.0898	0.1621	0.7684	0.8067	0.3142
Kurtosis	5.1438	3.5338	4.1411	4.9605	3.5987

Table 4.6: Statistics of simulated prices[7]

We perform the out-of-sample test with the data of the last month of 2004. We use each of the calibrated models to generate 5000 forecasted paths of daily prices for the out-of-sample month. For each generated path, two performance measures, namely Root Mean Squared Errors (RMSE) and Mean Absolute Errors (MAE), are calculated by

$$RMSE = \sqrt{\frac{1}{k}\sum_{i=1}^{k}[\ln(P_t) - \ln(P_t^f)]^2} \tag{4.40}$$

$$MAE = \frac{1}{k}\sum_{i=1}^{k} \mid \ln(P_t) - \ln(P_t^f) \mid \tag{4.41}$$

where k is the number of observations, P_t is the observed daily price in the forecast period, and P_r^f is the forecasted daily price.

The average forecast errors for each model are reported in Table 4.7. We find that the forecast errors of regime switching models are generally lower than those of MRJD model. After all, the De Jong-Huisman model outperforms all other models in forecasting the one-month ahead prices.

[7] Note that the historic price here include only the in-sample data which range from January 1, 2001 to November 30, 2004, thus the historic statistics in the second column of Table 4.6 are different from the second column of Table 4.1.

	MRJD	Mixture distribution	De Jong-Huisman	Ethier-Mount
RMSE	0.6960	0.5118	0.4621	0.4940
MAE	0.5553	0.4210	0.3850	0.3967
RMSE	0.6354	0.4758	0.4513	0.4921
MAE	0.4869	0.3952	0.3712	0.3824

Table 4.7: Average forecast errors of alternative models

4.5 Concluding Remarks

The electricity supply industry in many countries has transited from a regulated, monopolistic system to a deregulated and competitive market. The increasing trading activities in electricity arouse the need for electricity price modeling.

As a special commodity which is not economically storable, electricity prices exhibit several uncommon properties. Firstly, electricity prices are highly persistent to their long-term average level, which is termed as mean reversion. Secondly, electricity prices have remarkably high and time-varying volatilities. Thirdly, various seasonal patterns, such as intra-day, day of a week, and annual cycles, are found in electricity prices. Fourthly, price jumps occur occasionally in electricity markets. Finally, electricity price behaviors differ from one region to another.

With the reduced-form approach of electricity price modeling, different stochastic processes are exploited to reflect the above-mentioned price properties. We calibrate the two mostly used models – mean reverting jump diffusion and regime switching models – to the Dutch APX and German EEX market data. With a simulation-based approach, we compare the in-sample goodness-of-fit and out-of-sample forecasting accuracy between these two types of models. We find that the regime switching model, especially the De Jong-Huisman model gives a better performance.

The spot price modeling forms the foundation of derivatives pricing and risk management in the electricity markets.

In the following chapters, we will use these models to value energy assets as exotic derivatives contracts.

Chapter 5

Volumetric Risks and Power Plant Value

5.1 Introduction

Under a competitive market environment after the deregulation of electricity industry, an economically dispatching power plant is conveniently valued by a real options model, in which the value of the power plant is considered a string of spark spread call options.

Option-based valuation is able to capture the flexibility of operating a power plant. With an optimal operational schedule, revenues form positive spark spread values can be retained and losses from negative spark spread values can be avoided.

Electricity and fuels prices are key drivers of power plant values. We use two correlated stochastic processes to model these two prices and calibrate models to the historical data in the Dutch APX market.

Considering the complexity of the electricity industry, there are a variety of additional factors which can affect the realization of a power plant value. We classify these factors as volumetric risks, which may occur whenever a power plant encounters an unplanned outage, a sudden increase in electricity consumption, or a transmission failure.

Volumetric risks are related to the real-time nature in both production and consumption of electricity. Since electricity is hardly storable and demand of electricity is not elastic on price, volumetric matching will be a critical task for electricity producers and marketers. Volumetric risks are not caused by market price movements, but rather by physical problems. But the converse statement is true: Market price movements in power markets can all be tracked back to volumetric risks.

We classify volumetric risks into two categories – risks from the supply side and from the demand side. Risks from the supply side are mainly posed by the physical constraints in operating a power plant. Risks from the demand side come from the uncertainty in customer load.

Volumetric risks have a negative impact on the profit and consequently on the value of a power plant. This impact has been addressed in the literature of energy asset valuation with an option-based method. Examples can be found in [DJS01], [GZ00], [TB02] and [DO03]. However, in all above articles, the model only incorporates a few of physical constraints such as minimum up/down time, startup/shut-down costs, ramp-up time and varying heat rates. Furthermore, these constraints are treated in a highly simplified manner in their modeling.

This chapter follows the approaches used in [GZ00] and [TB02] by using backward dynamic programming based on Monte Carlo simulations. The contributions of this chapter to the literature are twofold. Firstly, we use regime switching models for electricity spot prices, which is capable of capturing the jumps in electricity prices. Secondly, we empirically assess the impacts of different volumetric risk factors on the power plant value, from both the supply side and the demand side.

This chapter is organized as follows. Section 5.2 discusses on different volumetric risk factors and how they may affect power plant values. In Section 5.3, we provide a general valuation framework using real options approach. In Section 5.4, we calibrate a regime switching model to electricity prices, a GBM model to gas prices, and a GBM model to customer load data. The changes in power plant value under different simulation scenarios are reported. Finally, Section 5.5 gives some concluding remarks.

5.2 Volumetric Risks and Their Impacts on Power Plant Values

In this section, we discuss various volumetric risk factor from both the supply side and the demand side. The impacts of these risk factors on power plant value are also analyzed in a qualitative way.

5.2.1 Impact of Supply-Side Risks and Constraints

On the supply side, a power plant is always subject to certain physical constraints which limit the production rate and/or operational flexibility. The final output level and power plant value are affected as well. These physical constraints include:

- Startup and shutdown costs. Startup and shutdown costs are used in calculating the cost of bringing a unit on or off line. There are two components for a start: a straight cost component and a fuel cost. These two may be dependent on whether the start is hot or cold.

- Minimum up/down time. Most generation units cannot be turned on and off as frequently as we expect. Every flexible unit may need to remain online for several hours before they can be shut down. Similarly, most units cannot be restarted immediately if the current status is off.

- Ramp rate. A generation unit has its maximum and minimum capacity output. The maximum capacity and minimum capacity also permit incorporation of environmental and/or season-dependent changes. Typically, a generation unit needs a certain length of time to move from zero MW to its full capacity. The ramp rate is a measure of how fast a generator can move up or down from its current state. Accordingly, we distinguish ramp-up rate and ramp-down rate. Ramp rate has a unit of MW/hour.

- Varying heat rate. The heat rate of a generation unit is not constant. As the output increases, the heat rate increases as well. Generally, a generating unit is most efficient when it is operated at or near its maximum capacity. Heat points represent the MW level used to define a heat rate.

- Forced outage. Generation units sometimes break down unexpectedly due to technical problems. Units lower in the commitment order pick up the swing duty shifted down by outages of base-load units. A forced outage rate represents the percentage of forced down hours to the whole number of hours in a year. For instance, a single forced outage rate of 10% means that the unit has a 90% probability of being available during any course of time. Forced outages are actually random events.

We only know they will happen with a certain probability but we do not know when exactly they will happen.

- Maintenance rate. Certain period of time is needed for the maintenance of a generation unit. Similar to a forced outage rate, a maintenance rate represents the percentage of maintenance hours to the whole number of hours in a year.

- Spinning reserve. A technical approach to maintain electricity supply security is to reserve some capacity of generation. The term of spinning reserve is used to describe the total amount of power available from all units "spinning" in the system minus the load that is supplied and losses that take place inevitably along the lines. Spinning reserves are allocated so that they obey certain rules. Normally, reserves should represent a given percentage of forecast peak demand, generally 10-20%; or it must be capable of making up the loss of the most heavily loaded unit in a given period of time. Another way of calculating the reserve requirement is to define it as a quantile of outage-related loss distribution: for instance, choose the reserve in such a way that the probability of not having sufficient generation is smaller than 0.01.

- Emission constraint. Environmental regulations require a generation unit to purchase emission right or invest in pollutant scrubbers. As emission right trading has been effective in Europe, the cost increase in producing electricity due to CO_2 prices will affect the output level and power plant value.

The above-listed characteristics have an impact on the power plant value in two different ways. The startup/shutdown cost, varying heat rate and emission constraint increase the strike price of the spark spread call options. For instance, with a higher heat rate, an additional cost is imposed to the exercising of the spark spread. Denote the change of heat rate as ΔK, then the additional cost if $\Delta K S_G$. If we write $D = \Delta K S_G$, then the payoff of the spark spread option at maturity time T is given by

$$C(S_E(T), S_G(T), K, T) = \max(S_E(T) - KS_G(T) - D, 0). \tag{5.1}$$

where the strike price of the call option increases from $KS_G(T)$ to $KS_G(T)+D$.

An increase in strike price implies a decrease in the call spark spread option. The power plant value will decrease as well.

The minimum up/down time, ramp rate, forced outage, maintenance and spinning reserve add a "swing" component to the spark spread options. These constraints limit the output rate or disable the production completely. In this way, the volume of the spark spread options at certain time, and consequently the value of the power plant, are decreased. For instance, at certain hours, a power plant with a maximum capacity of 100 MW may only have a output level of 80 MW due to spinning reserve requirement or ramping constraints. At some hours, the power plant has to be shut down for planned maintenance or by unexpected outage, then the realized volume of spark spread options drops zero.

5.2.2 Impact of Demand-Side Risk

An independent generator is not exposed to volumetric risk from the demand side. However, deregulation up to now has not created many independent generator. Major players in the markets are still those incumbents who own generating assets and service customer loads at the same time. We simplify the problem by studying a load-serving power plant. When taking customer load contracts into account, the power plant value is then the value of a portfolio, which consists of the embedded spark spread options and the aggregated customer load contract.

When valuing a load-servicing power plant, an additional complexity is the interaction among the generator, the spot power market and customer load. With a customer load constraint, the power plant does not only play against the spot market. The above-all rule is to meet the customer load at any time.

If the spot power price is greater than the power plant's dispatch cost, the power plant will run at its maximum capacity. In the running status, we still need to distinguish two scenarios. If this maximum capacity is greater than the customer load, the generator will sell the surplus output in the spot market. If the maximum capacity is lower than the customer load, the generator will buy the shortage electricity from the spot market.

In contrast, if the spot power price is lower than the power plant's dispatch cost, the power plant will be stop running and buy all the power from the spot market to satisfy the customer load. Note that we assume no impact on the spot price by individual buying and selling in the market.

Customer load is often assumed to follow a conventional pattern. The aggregated long-term load is likely to be increasing slowly with a time trend, due to the economic and population factors. However, the short-term customer load fluctuations, which are related to weather conditions, are difficult to predict.

The uncertainty in customer load affects the cash flows and as well values of a load-serving power plant. The impact is more complicated than the supply side physical constraints. To what extent the demand-side volumetric risk may impact the power plant value depends on the tariff system for end users. If the selling price of electricity to end customer is completely real-time, i.e., equals the spot market price, the uncertainty in customer load is then fully absorbed by the spot market and the related volumetric risk has no impact on the power plant value.

However, such a real-time tariff system is not applicable to end customers. Typically, customer load contracts specify a fixed forward price. These contracts are actually swing contracts. The power plant is exposed to the uncertain in customer loads.

One important property of customer load is its co-movement with prices. Note that the customer load always has a positive correlation with electricity price. This implies that the demand-side risk always works jointly with electricity prices. This multiplying effect of customer load uncertainty may pose remarkable risks on the power plant value[1].

5.3 Real Options Valuation

In order to price the spark spread, we model the electricity and the gas prices separately. Based on the conclusions in Chapter 4, we choose a regime switching model for electricity prices and a lognormal mean reverting model for gas prices. For the stochastic customer load, we use a lognormal mean reverting model as well. The Monte Carlo simulation of correlated processes is introduced.

[1] In the California power crisis in the summer of 2000, the volumetric risks coming from the demand side contributed to the bankruptcy of the Pacific Gas & Electric company.

5.3.1 Price and Load Models

Following the conclusions in Chapter 4, we choose to use a two-state regime switching model for electricity prices. The normal regime is given by a Geometric O-U process and the spike regime is governed by a GBM process. Furthermore, in order to reflect seasonal patterns in prices, all the parameters in these process are time-varying.

Since more than one variables are used here, we will rewrite the regime switching models from Chapter 4. The electricity price S_E is given by

$$S_E = S_E^1 \text{ or } S_E^2. \tag{5.2}$$

where S_E^1 represents electricity prices in a normal price regime and S_E^2 represents electricity prices in a jump regime. Assuming the price in the normal regime follows a mean reverting diffusion model and the price in the jump regime follows a lognormal distribution, the two regimes are then described by the following two processes:

$$d(S_E^1) = \kappa_E(t)(\mu_E^1(t) - \ln S_E^1)S_E^1 dt + \sigma_E^1(t)S_E^1 dW_E^1 \tag{5.3}$$

$$d(S_E^2) = \mu_E(t)S_E^2 dt + \sigma_E^2(t)S_E^2 dW_E^2 \tag{5.4}$$

where $\kappa_E(t)$ is the mean reverting rate of the logarithm of electricity price in the normal regime, $\mu_E^i(t)$ is the mean log price in regime i, $i = 1, 2$, $\sigma_E^i(t)$ is the standard deviation of electricity prices in regime i, $i = 1, 2$, dW_E^1 and dW_E^2 are the increments of two independent Wiener processes.

The transition possibilities from one regime to another is controlled by a Markovian matrix given by equation (4.29), i.e.,

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix} \tag{5.5}$$

where p_{ij} represent the probability of switching from regime i at time t to regime j at time $t + 1$, $i, j = 1, 2$.

Gas prices, S_G, are also mean reverting, but jumps are not often observed. Thus, we use a lognormal mean reverting model for gas prices. The process is given by

$$dS_G = \kappa_G(\mu_G(t) - \ln S_G)S_G dt + \sigma_G(t)S_G dW_G \tag{5.6}$$

where $\kappa_G(t)$ is the mean reverting rate of the logarithm of gas price, $\mu_G(t)$ is the mean log gas price, $\sigma_G(t)$ is the standard deviation of log gas prices,

dW_G is the increment of a standard Brownian which is correlated with dW_E^1 by

$$dW_E^1 dW_G = \rho_{E,G} dt. \tag{5.7}$$

When simulating a load-servicing power plant, we need a model for customer load. In practice, the prediction of a customer load is easier to be done than forecasting electricity prices, because customer load is stabler and its cyclical patterns are closely related the weather. We use a Geometric O-U model for customer load. The process is given by[2]

$$dL = (\mu_L(t) - \ln L) + \sigma_L dW_L \tag{5.8}$$

where $\mu_L(t)$ is the normal load level at time t, which is obtained from historical load profiles, σ_L is the standard deviation of logarithm of customer load, dW_L is the increment of a standard Brownian which is correlated with dW_E^1 by[3]

$$dW_E^1 dW_L = \rho_{L,E} dt. \tag{5.9}$$

5.3.2 Spark Spread and Power Plant Value

To recall some contents from earlier chapters, the holder of a European spark spread call option written on fuel G at a fixed heat rate K_H has the right but not the obligation to pay K_H times the unit price of fuel G at the option's maturity time T and receive the price of 1 unit of electricity. Let S_E^T and S_G^T be the spot prices of electricity and fuel at time T, respectively. Denote the value of the option at time t by $C(S_E^T, S_G^T, K_H, t)$, then we have

$$C(S_E^T, S_G^T, K_H, T) = \max(S_E^T - K_H S_G^T, 0) \tag{5.10}$$

Consider a power plant with a lifetime of $\bar{T}$, the value of the power plant V is then given by

$$V = \int_0^{\bar{T}} C(t)dt. \tag{5.11}$$

Following [GZ00] and [TB02], we choose to use a dynamic programing method which is based on Monte Carlo simulations. Under the electricity

[2] To simplify the estimation of $\rho_{E,G}$, we assume gas prices are only correlated with the electricity prices in the normal regime.

[3] To simplify the estimation of $\rho_{L,E}$, we assume customer loads are only correlated with the electricity prices in the normal regime.

and gas price (and customer) paths generated by simulations, the optimal operational schedule for a generation unit is solved from the terminal condition backward until to time zero[4]. The final power plant value is then the average of all the power plant values realized with each optimal schedule under each simulated paths.

In the next subsection, we will introduce the Monte Carlo simulation method for correlated processes.

5.3.3 Monte Carlo Simulation

Monte Carlo simulations generate random paths of price series in the forecasting time period. If electricity prices are given by its normal regime, the spark spread value is then driven by the following two mean reverting lognormal processes.

$$dS_E = \kappa_E(\mu_E(t) - \ln S_E)S_E dt + \sigma_E(t)S_E dW_E^t \quad (5.12)$$
$$dS_G = \kappa_G(\mu_G(t) - \ln S_G)S_G dt + \sigma_G(t)S_G dW_G^t \quad (5.13)$$
$$dW_E^t dW_G^t = \rho_{E,G}dt \quad (5.14)$$

In this subsection, we show the Monte Carlo simulations with these two correlated mean reverting processes. The procedure is the same if the electricity price is given by its spike regime. The price-correlated customer load can be so simulated as well.

Following [Gem05], we construct two correlated Brownian motions. Using Cholesky decomposition method, these two Brownian motions can be written as

$$W_G^t = \rho_{E,G}W_E^t + \sqrt{1-\rho_{E,G}^2}W_I^t \quad (5.15)$$

where W_I' is a Brownian motion independent of W_E^t.

By dividing the time to maturity $[0, T]$ into n subintervals, the changes in S_E and S_G over the first interval $[0, T/n]$ are defined by the draws of the Brownian increments:

$$H_E = W_E(0 + \frac{T}{n}) - W_E(0) \quad (5.16)$$
$$H_G = W_G(0 + \frac{T}{n}) - W_G(0) \quad (5.17)$$

[4]The total number of possible spark spread options depends on the technical assumptions we make for the power plant. In this chapter, we assume the operator of the peak-load power plant makes its decision at each hour.

The discretization of the electricity and gas price processes are

$$S_E(\frac{T}{n}) = \kappa_E(\mu_E(\frac{T}{n}) - \ln S_E(0))S_E(0)\frac{T}{n} + \sigma_E(\frac{T}{n})S_E(0)H_E \quad (5.18)$$

$$S_G(\frac{T}{n}) = \kappa_G(\mu_G(\frac{T}{n}) - \ln S_G(0))S_G(0)\frac{T}{n} + \sigma_G(\frac{T}{n})S_G(0)H_G \quad (5.19)$$

The quantity of H_E is normally distributed with mean 0 and variance T/n. In order to make a random pair of S_E and S_G, we first create a random draw of $\hat{H}_E$ following this normal distribution. Then over the same time interval, we obtain $\hat{H}_G$ by building another random draw which is independent of W_E^t and has the same distribution: mean 0 and variance T/n. Then we have

$$\hat{H}_G = \rho_{E,G}\hat{H}_E + \sqrt{1-\rho_{E,G}^2}\hat{H}_I \quad (5.20)$$

Constructing M pairs of trajectories for S_E and S_G in this manner, we can obtain the corresponding terminal values of $S_E(T)$ and $S_G(T)$. Figure 5.3 and Figure 5.4 show one path of the simulated electricity and gas price, respectively. As described in subsection 5.3.2, under each pair of generated price paths, dynamic programming gives an optimal operational schedule and realizes a power plant value u_i, $i = 1, 2, ..., M$. The Monte Carlo simulated power plant value V^{MC} is then equal to

$$V^{MC} = \frac{1}{M}\sum_{i=1}^{M} u_i. \quad (5.21)$$

5.4 Empirical Results

In this section, we calibrate the models specified in Section 5.2 and use simulations to test the impacts of volumetric risk factors on power plant values. We run the simulations under various scenarios, in which different risk factors are introduced, and compare the changes in power plant values.

5.4.1 Data and Model Calibration

We assume a virtual gas-fired peak power plant operated in the Dutch markets with a permanent capacity of 100 MW. The average heat rate of 7500 Btu/KWh. The power plant is valued on February 10, 2006, and is supposed to operate from January 1 to December 31, 2007.

Using data sample from July 1, 2003 to February 10, 2006, we calibrate the price models of electricity to the APX price data, the gas price model to TTF 51.7 gas prices, and load model to a typical historical load profile.

We do not conceive CO_2 trading as a structural change in the markets. As an alternative, we view the emission cost as a constraint which is explicitly taken into account in the model.

In order to model the seasonal patterns in both commodities, we divide the total data samples into four seasons, i.e., Spring (from March to May), Summer (from June to August), Fall (from September to November), and Winter (from December to February of next year). We further divide the data in each season into two sections, i.e., weekdays and weekends. We then estimate the parameters for each subsection accordingly.

We do not assume the weekday/weekend patterns for gas prices, because they are not obviously observed in the markets. Totally, we obtain $4 \times 2 = 8$ sets of parameters for electricity prices and 4 sets of parameters for gas prices. The correlations between the two commodities' prices are also estimated in each subsection sample. The estimated parameters are offered in Table 5.1.

Panel 1: Parameters for weekdays

Electricity price model

Season	μ_E^1	μ_E^2	κ_E^1	σ_E^1	σ_E^2	p_{11}	p_{22}
Spring	3.4649	3.8557	0.3241	0.1219	0.2204	0.7356	0.2549
Summer	3.3921	5.1945	0.6281	0.3166	1.0270	0.9681	0.6445
Fall	3.1451	4.0826	0.8984	0.2515	0.3450	0.8542	0.5663
Winter	3.4678	3.8557	0.5799	0.2724	0.4907	0.9783	0.8399

Gas price model

Season	κ_G	μ_G	σ_G
Spring	0.2476	2.3856	0.0338
Summer	0.0099	2.3444	0.0341
Fall	0.1371	2.5486	0.0553
Winter	0.0492	2.5362	0.0392

Price correlation

Season	$\rho_{E,G}$
Spring	0.1743
Summer	0.2231
Fall	0.1739
Winter	0.4431

Panel 2: Parameters for weekends

Electricity price model

Season	μ_E^1	μ_E^2	κ_E^1	σ_E^1	σ_E^2	p_{11}	p_{22}
Spring	2.8923	3.4393	0.1859	0.2154	0.1420	0.2931	0.7387
Summer	3.2560	3.4209	0.6655	0.3449	0.0675	0.2923	0.1971
Fall	3.2458	3.6708	0.8579	0.1425	0.2898	0.2685	0.7665
Winter	3.3052	3.5461	0.7439	0.4420	0.1115	0.3651	0.6267

Gas price model

Season	κ_G	μ_G	σ_G
Spring	0.2476	2.3856	0.0338
Summer	0.0099	2.3444	0.0341
Fall	0.1371	2.5486	0.0553
Winter	0.0492	2.5362	0.0392

Price correlation

Season	$\rho_{E,G}$
Spring	0.1825
Summer	0.0725
Fall	0.1123
Winter	0.2573

Table 5.1: Parameters estimation results (Note that all volatilities in this table are on a daily basis)

5.4.2 Electricity Hourly Price Index

Unlike gas prices, the electricity prices are not flat during one day. The day-ahead prices in APX are quoted on a hourly basis. The peak-load power plant in our model is assumed that the dispatch decisions can be made in every hour. Thus, we need a method to translate the simulated daily prices into hourly prices for dispatch decision-making and cash flow calculation.

We construct a multiplicative hourly index system based on historical hourly prices from July 1, 2003 to February 10, 2006. The estimated hourly price index for each hour is just the average weight of the price of that hour in the daily average price. In order to represent price patterns, we estimate $4 \times 2 = 8$ sets of hourly price indexes. For example, the hourly indexes for the weekdays in spring are estimated by

$$\tilde{H}_{i,t} = \frac{1}{m} \sum_{t=1}^{m} \phi_{spring} \eta_{weekday} P_{i,t} \tag{5.22}$$

where $\tilde{H}_{i,t}$ is the estimated price index for the i-th hour ($i = 1, 2, ..., 24$) on date t, m is the number of observations, $P_{i,t}$ is the observed hourly price at the i-th hour ($i = 1, 2, ..., 24$) at date t, ϕ_{spring} and $\eta_{weekday}$ are dummy variables, and $\phi_{spring} = 1$ when date t is in spring and $\phi_{spring} = 0$ otherwise, $\eta_{weekday} = 1$ when date t is a weekday and $\eta_{weekday} = 0$ otherwise.

The hourly price indexes are used as multipliers to the average daily price, reflecting the intra-day price patterns. We also find that the indexes differs between weekdays and weekend, and vary across different seasons. These properties of hourly price patterns can be more easily observed in Figure 5.1 and Figure 5.2. We find that the two peaks during a day are more pronounced in winter and summer. These hours are important for peak-load power plants.

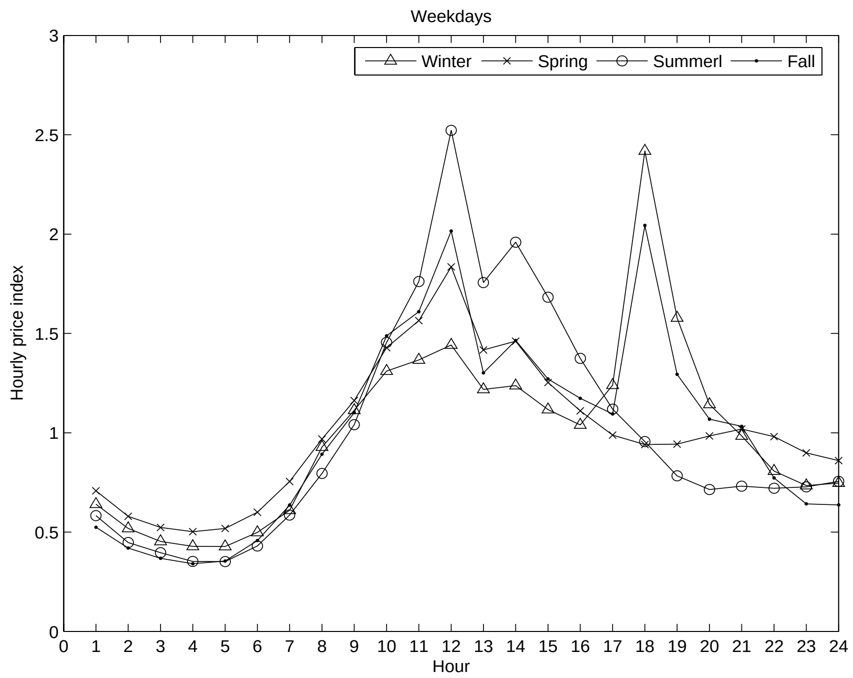

Figure 5.1: Hourly price indexes in weekdays

With the hourly multiplicative indexes for each hour, we can always work with daily average prices. In case we need to forecast the hourly prices for dispatch planning, we can convert the daily average prices into hourly prices by multiplying the correspondent multiplier for each hour, i.e.,

$$P_{i,t} = \tilde{H}_{i,t} P_t \tag{5.23}$$

where P_t is the daily average price.

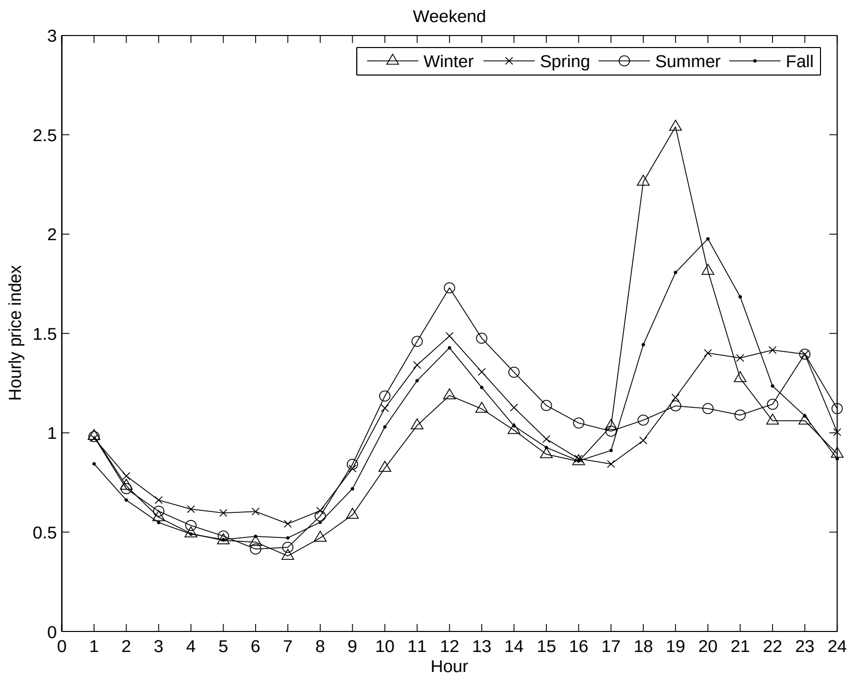

Figure 5.2: Hourly price indexes in weekend

5.4.3 Simulations With Supply-Side Risk Factors

Based on the calibrated electricity and gas price models, we use a Monte Carlo simulation engine, Planning and Risk (PAR) to generate possible price paths and power plant generation strategies [Glo04]. The generated daily average electricity prices are in converted into hourly prices by multiplying the hourly price indexes. The hourly prices of gas are assumed to be flat during each day. The net present values (NPVs) of the power plant are calculated by discounting the future cash flows with a risk-adjusted discount rate. We use an annual discount rate of 10%[5] and discount the future cash flows on a monthly basis.

We perform the simulations under various scenarios. For the base case scenario, we assume no physical constraint for the power plant, and we use the average heat rate. Next, we construct six other scenarios by adding

[5] This rate is considered exogenous here. A Weighted Average Cost of Capital (WACC) model is often used to decide this rate.

one supply-side volumetric risk factor each time, successively. Under each scenario, we run Monte Carlo simulations with $N = 350$ iterations[6] to obtain the value of the power plant. Specifically, these six scenarios are

1. +Minimum up time of 8 hours and minimum down time of 2 hours;
2. +Ramp rate of 30 MW/hour from capacity of 40 MW to 100 MW;
3. +Varying heat rate defined by heat points 40, 60, 80 MW and heat rates 8500, 7500, 6500 Btu/KWh, respectively;
4. +Forced outage rate of 0.05 and maintenance rate of 0.04;
5. +Spinning reserve rate of 0.1 and the spinning reserve provider receives a revenue which is 20% of the spot electricity price;
6. +Startup/shutdown cost of 1000 Euro per start(shut) and emission cost of CO_2 15 Euro/ton with an emission rate of 350g/KWh.

The above constrains are so chosen that they can reflect realistic power plants. Simulations based on these constraints should demonstrate the major effects of the risk factors.

The dispatch of the power plant is determined by a dynamic programing method. This method requires that at each time point the dispatch action should maximize the current cash flow plus the expected cash flow for the remaining future time.

We introduce the following notations:

- t: time index ($t = 0, 1, 2, ..., T$);
- x_t: state variable with its sign indicating whether the plant is started up (+) or shut down (-) and its magnitude indicating the length of the time being in this mode;
- u_t: binary decision variable equals 1 when the decision is to run and 0 when the decision is to shut down;
- q_t: decision variable denoting the generating capacity at time t;

[6]The PAR simulation engine uses an antithetic sampling technique. Our experiements show that 350 paths suffices to make the power plant value to converge.

- HR_t: effective heat rate at time t;
- $C_{o,t}$: outage cost at time t;
- $C_{m,t}$: maintenance cost at time t;
- $C_{u,t}$: start-up cost at time t;
- $C_{d,t}$: shut-down cost at time t;
- $C_{e,t}$: emission cost at time t;

The six scenarios are then translated into:

1. $u_t = \begin{cases} 1, \text{ if } 1 \leqslant x_t \leqslant 8 \\ 0, \text{ if } -2 \leqslant x_t < 1 \\ 0 \text{ or } 1, \text{ otherwise} \end{cases}$

   $x_t = \begin{cases} x_{t-1} - 1, \text{ if } x_{t-1} \in (-2,-1), \text{ or, } x_{t-1} \in (-\infty,-2] \text{ and } u_t = 0; \\ 1, \text{ if } x_{t-1} \in (-\infty,-2] \text{ and } u_t = 1; \\ x_{t-1} + 1, \text{ if } x_{t-1} \in [8,+\infty), \text{ or, } x_{t-1} \in (8,+\infty) \text{ and } u_t = 1; \\ 1, \text{ if } x_{t-1} \in (8,+\infty) \text{ and } u_t = 0 \end{cases}$

2. $40 \cdot sign(x_t) \leqslant \mid q_t - q_{t-1} \mid \leqslant 100 \cdot sign(x_t)$

   where $sign(x_t) = \begin{matrix} 1, \text{ if } 0 < x_t; \\ 0, \text{ otherwise.} \end{matrix}$

3. $HR_t = \begin{cases} 6500 \times \frac{0.2931}{1000}, \text{ if } q_t \geqslant 80; \\ 7500 \times \frac{0.2931}{1000}, \text{ if } 60 \leqslant q_t < 80; \\ 8500 \times \frac{0.2931}{1000}, \text{ if } 40 \leqslant q_t < 60; \end{cases}$

4. $C_{o,t} = \begin{cases} +\infty, \text{ if } out_t = 1, \text{ where } \Pr(out = 1) = 0.05; \\ 0, \text{ otherwise.} \end{cases}$

   $C_{m,t} = \begin{cases} +\infty, \text{ if } maintain_t = 1, \text{ where } \Pr(maintain = 1) = 0.04; \\ 0, \text{ otherwise.} \end{cases}$

5. $C_{r,t} = \begin{cases} (1 - 0.2) S_E^t, \text{ if } x_t > 0; \\ 0, \text{ otherwise.} \end{cases}$

6. $C_{u,t} = \begin{cases} 1000, \text{ if } x_t > 0 \text{ and } x_{t-1} < 0 ; \\ 0, \text{ otherwise.} \end{cases}$

$C_{d,t} = \begin{cases} 1000, \text{ if } x_t < 0 \text{ and } x_{t-1} > 0 ; \\ 0, \text{ otherwise.} \end{cases}$

$C_{e,t} = \begin{cases} \frac{15\times 350}{1000} \cdot q_t, \text{ if } x_t > 0; \\ 0, \text{ otherwise.} \end{cases}$

Denote the cash flow obtained at time t as f_t, then

$$f_t(x_t, u_t, S_E^t, S_G^t) = S_E^t q_t - HR_t S_G^t - C_{o,t} - C_{m,t} - C_{u,t} - C_{d,t} - C_{e,t} \quad (5.24)$$

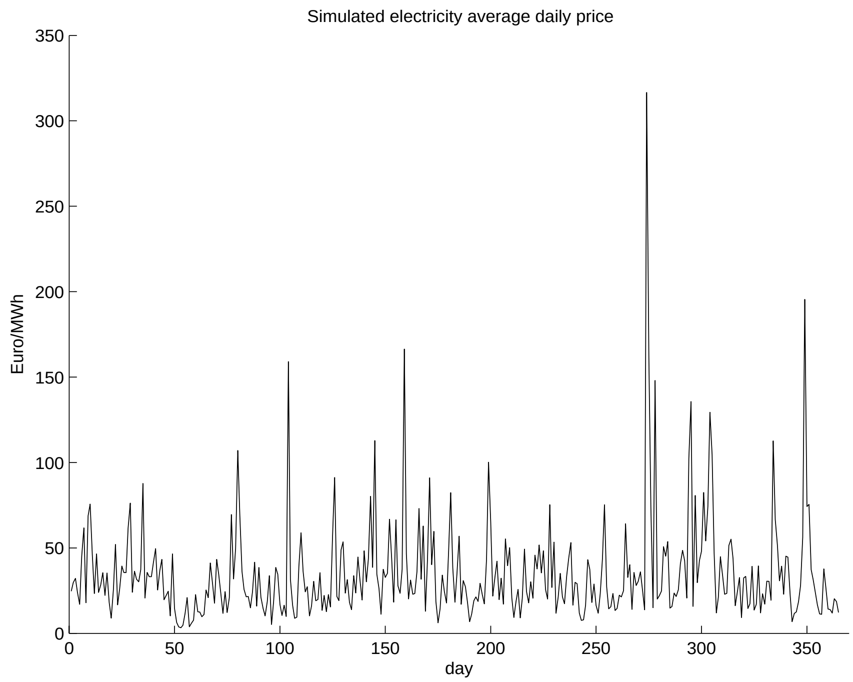

Figure 5.3: Simulated electricity prices

Denote the power plant value at time t as V_t, with r being the constant discount rate, then

$$V_t(x_t, u_t, S_t^E, S_t^G) = \frac{f_t(x_t, u_t, S_t^E, S_t^G)}{(1+r)^t} + \max_{\tilde{u}_t} E_t[V_{t+1}(x_t, \tilde{u}_t, S_t^E, S_t^G)] \quad (5.25)$$

where E_t denotes the expectation operator given the information at time t. The Bellman equation is subject to the correspondent constraint scenarios.

For the i-th pair of simulated price paths for electricity and gas, the Bellman equation can be solved recursively from the ending time $t = T$ backward until $t = 0$. Then V_0^i is the power plant value for the i-th price paths. Finally, the estimated power plant value $V_0 = \frac{1}{N}\sum_{i=1}^{N} V_0^i$.

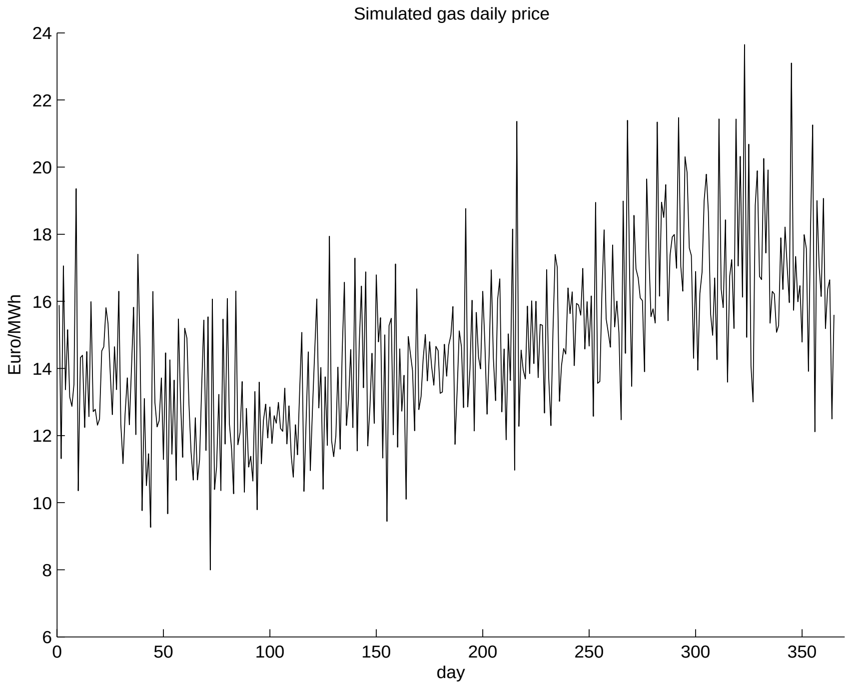

Figure 5.4: Simulated gas prices

The simulation-based power plant values under different scenarios are plotted in Figure 5.5. We find that the power plant values decreases obviously with the inclusion of physical constraints. Especially, the forced outage rate, the maintenance rate and spinning reserve have a pronounced effect in decreasing the power plant value. These findings are consistent with the roles the supply-side volumetric risk factors play in deciding the spark spread option values. Either the increases in the strikes or the decreases in the volume of the spark spread options tends to decrease the option-based power plant value.

For each pair of price paths under each scenario, the optimal generation schedule for the power plant are obtained. This optimal schedule makes little practical sense, because we have too many pairs of price paths for simulation but we don not know which pair of path will exactly predict the prices in the future. For this reason, the optimal schedules are not reported here.

5.4.4 Simulations With Demand-Side Risk Factors

In order to study of the demand-side risk and its impact on the power plant value, we start with the base case scenario which has been defined in Subsection 5.4.3. We use the same calibrated electricity and gas price models, the same specifications for the power plant as in Subsection 5.4.3.

We add two components for the simulations for the demand-side risk factors. The first one is the customer load. The second one is the fixed price specified in customer forward contracts. We use the customer load historical data from January 1, 2003 to Mar 31, 2006 to estimate the parameters of the stochastic load model.

We estimate the impact of demand-side risk by comparing the power plant values under a deterministic and a stochastic customer load. A deterministic customer load is given by the pattern described by the solid line in Figure 3. The average load during the modeling period is 51.64 MW.

Then, we rerun the simulations under the base scenario for both a deterministic and a stochastic customer load. As shown in Figure 5.5, the simulated stochastic customer loads exhibit moderate deviations from its deterministic path, with the 90% confidence levels illustrating the load fluctuations.

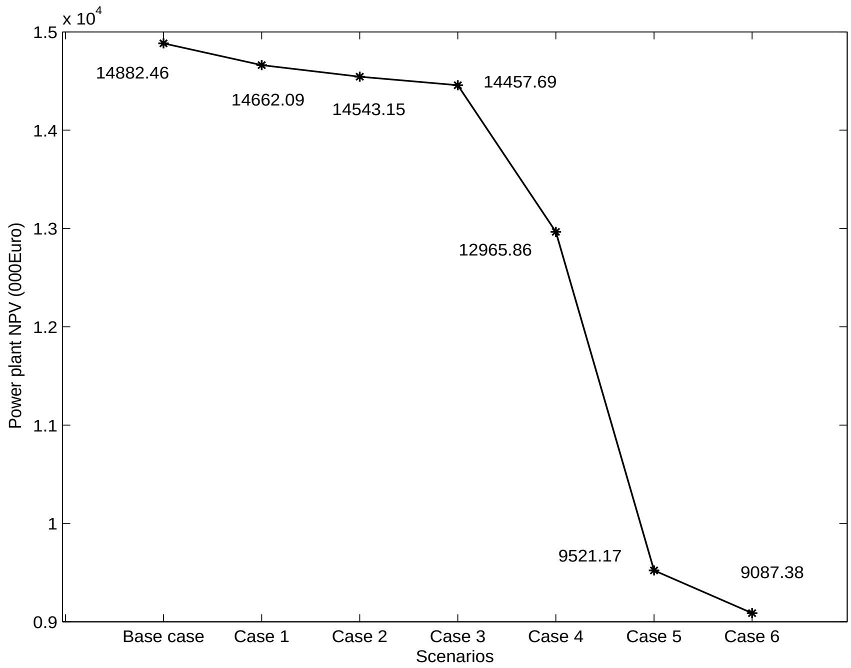

Figure 5.5: Power plant NPVs (in 000 Euro) under various volumetric risk scenarios

Since an economic dispatching method is applied, by adding the customer load, the power plant should interact with the spot market by selling the excess or buying the shortage electricity.

In practice, a load-servicing power plant often uses a fixed price in forward contracts with its end customers. This forward charge price is based on the forecast of electricity prices in the future.

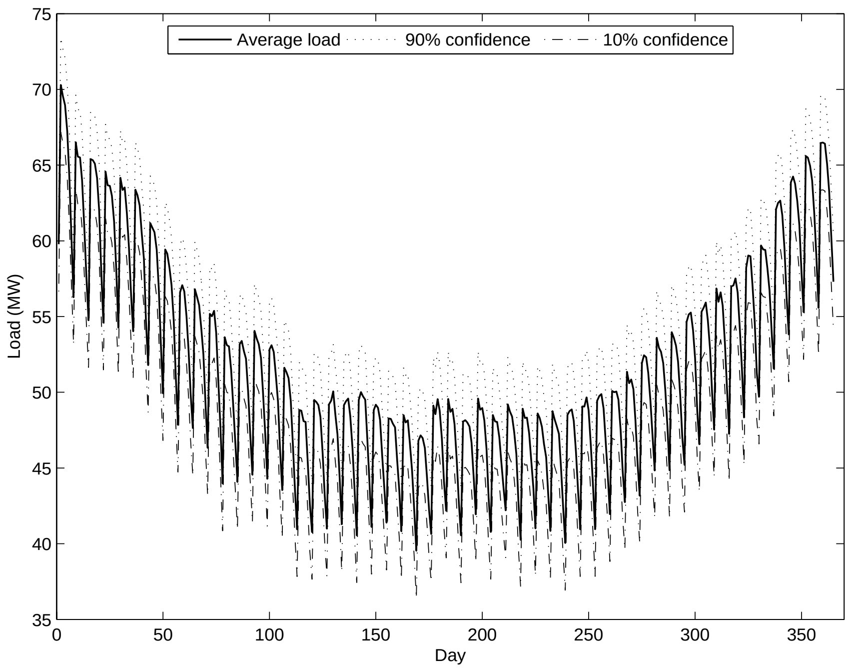

Figure 5.5: Deterministic and stochastic customer load

Since the revenue from servicing end-use customer loads is largely dependent on fixed price in forward contracts, we specify this fixed price to be 20, 36, 50, or 80 Euro/MWh to build up different simulation scenarios. Among these four fixed prices, 36 Euro/MWh equals to the average forecasted forward prices from the calibrated model. For each of the four fixed prices for the forward contracts, we run two parallel simulations – one with a deterministic customer load, one with a stochastic customer load. We find that the power plant value with a stochastic customer load is always higher than the value under by altering the forward charge price widely, the absolute loss in the power plant value a deterministic customer load. The percentage changes in power plant value are reported in Table 5.2.

The decreases in power plant value in Table 5.2 imply the negative impacts of the customer load uncertainty. The demand-side risks cannot be fully covered if the fixed price in forward customer contracts is higher than the average of forecasted electricity forward prices. Obviously, other types of contracts are needed to hedge the demand-side risk.

Fixed price in forward contracts	NPV changes
(Euro/MWh)	(%)
20	-12.83
36	-6.57
50	-2.32
80	-0.16

Table 5.2: NPV changes from a deterministic to a stochastic customer load

5.5 Conclusions

Together with the price uncertainties in electricity and fuels, the volumetric risks play an important role in determining the value of a power plant.

Volumetric risk factors come from both the supply and demand side in the electricity industry. These risks decrease the spark spread call option value by either increasing the strike prices or adding swing components to the option.

Volumetric risks from the supply side are mainly determined by the physical constraints in power plants. Our empirical simulation results show that, some risk factors, such as the forced outage rate, the maintenance rate and spinning reserve have a pronounced effect in decreasing the power plant value.

Due to the positive correlations between customer load and electricity prices, the demand-side risk always impacts power plant values jointly with price risks. This multiplying effect may pose a remarkable damage on power plant value. Fixed-price forward customer contracts, even if the fixed price is set higher than the average forecasted forward prices, cannot fully hedge the demand-side risk.

In the next chapter, we will study the investment opportunities in power plants.

Chapter 6

Power Plant Value and Investment Decisions

6.1 Introduction

The electricity supply industry is well-known as a capital-intensive sector. Currently, the high capital investment in power generation is further motivated by economic growth in the world, environmental compliance to the Kyoto Protocol, and competitive strategies of energy companies. According to the International Energy Agency report, the total investment requirement for the electricity supply sector worldwide over the period of year 2001-2030 will amount to 10 trillion US dollars, of which 4.5 trillion US dollars will be spent on power generation. New capacity of 4700 GW of the new generation (of which 2000 GW will be gas-fired) will cost over 4 trillion US dollars [IEA03].

With the introduction of competitive electricity markets, power generation investment analysis has become an important issue for electricity companies. This is not only because huge amount of capital is involved in power generation investment projects, but also for the reason that the rights to invest in new generation capacity are usually determined by a licence auction offered by the government. Power companies must compete for the investment opportunities by bidding for the licence and a central question for the companies is what price they should bid for such a licence.

The real options method fits well into the valuation of power plant investment opportunities. Firstly, as long as the operational flexibility of a power plant is considered, the power plant can be modelled as real options. While a base-load power plant can be regarded as a string of forward contracts, a peak-load power plant can be regarded as a string of call options on spark spreads. Examples of valuing a peak power plant or generation unit with real options method include [DJS01], [GZ00], [TB02], and etc. In Chapter 5, we demonstrated the valuation of a peak-load power plant with a rich range of operational characteristics.

Secondly, the opportunities to invest in power plants can also be modeled and valued as real options. Research along this line falls into the classic real options valuation framework [DP94] [Tri96], but is recent and rare, due to the short history of electricity and gas trading. Available literature includes [NF04a], [NF04b], [AC05] and [AC06].

The spark spread is the main value driver of a power plant, but it is not directly observable in the markets. We can only observe electricity prices and gas prices in the markets. For power plants with different efficiency (heat rate), we have different spark spreads. When valuing a power plant value or power plant investment opportunities, we model normally the electricity prices and gas prices separately and calculate the forecasted cash flows with certain heat rates. This approach is used in [DJS01], [TB02], [GZ00] and Chapter 5 of this thesis.

An alternative approach is to take the spark spread as the underlying. The spark spread is thought to be "observable" from the markets under a constant heat rate. If we assume an average efficiency (heat rate) for an invested power plant, the spark spread series can then be formulated and be used for power plant valuation. This approach is adapted by [NF04a] and [NF04b]. It has two advantages. Firstly, a one-factor model for the spark spread is as efficient as a two-factor model considering both electricity and gas prices. Modeling spark spread directly can avoid the estimation and modeling of the correlation between electricity and gas prices. Secondly, due to the similar seasonal patterns for electricity and gas prices, the seasonal effect in spark spread is largely mitigated by the offsetting of seasonalities in electricity and gas prices. For this reason, we can ignore the seasonal factors when working with spark spread processes.

[NF04a] derive the valuation formulas for both base-load and peak-load power plants. By using the short-term long-term two-factor model [SS00] for spark spreads, they assume investment decision is only determined by the long-term equilibrium spark spread. This reduces to a real options problem with the underlying variable following an Arithmetic Brownian motion process. In a classic real options framework, the theoretical thresholds to invest in a base-load power plant, to upgrade a base-load into a peak-load power plant, can be derived. However, the long-term equilibrium spark spread prices in the model are not observable in the markets and can only be estimated from electricity and gas forward prices. Furthermore, in the numerical examples, [NF04a] estimate the short-term parameters with short-maturity forward prices, and parameters for the equilibrium spark spread with the long-maturity forward prices. This manipulation distorts the two-factor model from [SS00] and leads to the result that the investment is simply determined by long-maturity forward prices.

This chapter follows the spirit of [NF04a] and [NF04b] by working on the constructed spark spread prices. However, we take a new approach by working on the historical spark spreads rather than the forward prices, since the exchange-quoted gas forward prices have just begun to be available in the Benelux markets. We carry out an empirical analysis on the spark spread prices in the Dutch markets. It is seen that the spark spread prices are less volatile than electricity prices, mean reverting, have less obvious seasonalities and can have negative prices. These findings motivate us to use a one-factor mean reverting process for spark spreads.

A major contribution of this chapter is the derivation of closed-form formulas for both base- and peak-load power plant values, given the spark spread following a one-factor mean reverting model.

A second contribution of this chapter is that we give a complete answer to the central question asked by power companies about investment licence valuation. As a general answer, the value of an investment licence is equal to the value of options embedded in the licence. [NF04a] study the option to upgrade a base-load into a peak-load power plant but only focuses on the equilibrium price threshold for upgrading. [NF04b] consider an option to abandon a power plant and finds that the option to abandon is of trivial value while the option to wait and operational flexibility of a power plant are of major importance. [AC06] valuate an option to double a power plant capacity, but only the base-load power plant is considered. In this chapter, we compute various investment options, including investment options in a peak-load power plant, where compound options are involved. We consider the investment options with various maturities: not only the now-or-never investment option, but also perpetual options and options with finite-time maturities. We value one most important option in power plant investment – the option to expand. We use both the Hull and White tree-building method and the Least Squares Monte Carlo (LSMC) method to value these exotic options. We also estimate the market price of risk and study its impact to the option values.

The remainder of this chapter is organized as follows. Section 6.2 describes the one-factor mean reverting model for spark spreads and derives the formulas for base- and peak-load power plant values. Section 6.3 examines the historical spark spread series in the Dutch markets and performs the parameter estimation. Section 6.4 analyzes the sensitivities of power plant values to the variation of different parameters in the spark spread model. In Section 6.5, we value the American option to invest in a base- or peak-load power plant. Section 6.6 values the American option to invest in an expandable base- or peak-load power plant. Section 6.7 estimates the market price of risk for the spark spread with the forward curve on the reporting day and discusses the impacts on the option value and investment decisions. Section 6.8 concludes this chapter and gives suggestions for some future research directions.

6.2 Spark Spread Model and Power Plant Value

For simplicity, we assume the energy companies make their investment decisions according to exogenous spark spread prices. As observed in an empirical investigation in the next section, the spark spread prices exhibit a strong mean reversion and have the possibility of being negative. Hence, we assume that the spark spread prices follow an Ornstein-Uhlenbeck stochastic process, which is given by

$$dX(t) = \kappa[\alpha - X(t)]dt + \sigma dZ(t) \tag{6.1}$$

where $X(t)$ is the spot spark spread price with a unit of Euro/MWh, α is the equilibrium spark spread value, κ is the mean reversion rate with which the $X(t)$ is pulled toward α, σ is the global volatility of the spark spread series, and $Z(t)$ is a Brownian motion.

Conditional on the initial realization of $X(0)$, the value of the spark spread at a future date T can be written as

$$X(T) = e^{-\kappa T}X(0) + (1 - e^{-\kappa T})\alpha + \sigma e^{-\kappa T}\int_0^T e^{\kappa t}dZ(t) \tag{6.2}$$

We find that $X(T)$ is normally distributed, and its expected value is given by

$$E_0[X(T)] = e^{-\kappa T}X(0) + (1 - e^{-\kappa T})\alpha \tag{6.3}$$

and the variance of $X(T)$ is

$$Var_0[X(T)] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa T}) \tag{6.4}$$

Assume that the variable cost (excluding fuel cost) of generating one unit of electricity with a base-load power plant, G_B, is constant through its lifetime. With a capacity of Q and a capacity factor[1] of ξ, the instantaneous cash flow generated by operating this base-load power plant is then

$$D(t) = Q\xi[X(t) - G_B] \tag{6.5}$$

The future cash flow at date T , conditional on $D(0)$, can be written as

$$D(T) = e^{-\kappa T}D(0) + Q\xi(1 - e^{-\kappa T})(\alpha - G_B) + Q\xi\sigma e^{-\kappa T}\int_0^T e^{\kappa t}dZ(t) \tag{6.6}$$

We find that $D(T)$ is normally distributed, and its expected value is

$$E_0[D(T)] = e^{-\kappa T}D(0) + Q\xi(1 - e^{-\kappa T})(\alpha - G_B) \tag{6.7}$$

and the variance of $D(T)$ is

$$Var_0[D(T)] = \frac{(Q\xi\sigma)^2}{2\kappa}(1 - e^{-2\kappa T}) \tag{6.8}$$

Under the risk-neutral measure, we assume that the market price of one unit of diffusion risk $Z(t)$ is a constant λ. The risk-neutral dynamics of the spark spread is given by

$$dX(t) = \kappa[\alpha^* - X(t)]dt + \sigma dZ^*(t) \tag{6.9}$$

where

$$\alpha^* = \alpha - \frac{\sigma\lambda}{\kappa} \tag{6.10}$$

$$dZ^*(t) = dZ(t) + \lambda dt \tag{6.11}$$

[1] The capacity factor describes the efficiency of a power plant. A factor of ξ means that over the whole life of the power plant, $(1 - \xi)$ of the time will be not possible to run.

The market price of risk λ is a measure of the extra return, or risk premium, that investors demand to bear risk. It is defined by the extra return divided by the amount of risk. In order to value derivatives, the risk-neutral process for the underlying is necessary.

Following the Girsanov Theorem, there exists a probability measure ρ^*, equivalent to the real probability measure ρ [Bjo04], such that the process

$$Z_t^* = Z_t + \int_0^t \lambda(s)ds = Z_t + \lambda t$$

equals Brownian motion under the measure ρ^*.

Then the future value of the spark spread at time T, $X(T)$, conditional on the initial realization $X(0)$, can be written as

$$X(T) = e^{-\kappa T}X(0) + (1 - e^{-\kappa T})\alpha^* + \sigma e^{-\kappa T}\int_0^T e^{\kappa t}dZ^*(t) \quad (6.12)$$

We denote $\mu^*(t)$ as the expected value of $X(T)$ at time t and $(\sigma^*(t))^2$ as the variance of $X(T)$ at time t under the risk-neutral measure, then we have

$$\mu^*(t) = E_t^*[X(T)] = e^{-\kappa(T-t)}X(t) + (1 - e^{-\kappa(T-t)})\alpha^* \quad (6.13)$$

$$(\sigma^*(t))^2 = Var_t[X(T)] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(T-t)}) \quad (6.14)$$

In consistency with the Girsanov Theorem, the variance remains unchanged under the risk-neutral measure.

Next, we come to the cash flow of a base-load power plant. Using a risk-neutral measure, the future cash flow at date T, conditional on $D(0)$, can be written as

$$D(T) = e^{-\kappa T}D(0) + Q\xi(1 - e^{-\kappa T})(\alpha^* - G_B) + Q\xi\sigma e^{-\kappa T}\int_0^T e^{\kappa t}dZ^*(t) \quad (6.15)$$

where

$$\alpha^* = \alpha - \frac{\sigma\lambda}{\kappa} \quad (6.16)$$

$$dZ^*(t) = dZ(t) + \lambda dt \quad (6.17)$$

$D(T)$ is still a normally distributed process, and its expected value is written as

$$E_0^*[D(T)] = e^{-\kappa T}D(0) + Q\xi(1 - e^{-\kappa T})(\alpha^* - G_B) \quad (6.18)$$

and the variance of $D(T)$ is

$$Var_0^*[D(T)] = \frac{(Q\xi\sigma)^2}{2\kappa}(1 - e^{-2\kappa T}) \tag{6.19}$$

Again, only the expected value of $D(T)$ is changed. The variance of $D(T)$ remains the same as under the original measure.

Assume that the risk-free interest rate r is constant during the whole time interval. The present value (at time 0) of receiving an uncertain payoff $D(T)$ at some future time T is

$$\begin{aligned} V_0[D(T)] &= E_0^*[e^{-rT} D(T)] \\ &= e^{-rT} E_0^*[D(T)] \\ &= e^{-(r+\kappa)T} D(0) + Q\xi(e^{-rT} - e^{-(r+\kappa)T})(\alpha^* - G_B) \end{aligned} \tag{6.20}$$

Then the value of the base-load power plant with a life time of $\overline{T}$ can be described as the value of receiving the continuous dividend cash flow $D(t)dt$ from date 0 through the future date $\overline{T}$. The base-load power plant value, V_B, can be obtained by integrating equation (6.20) with regard to t over the time interval $(0, \overline{T})$, i.e.,

$$\begin{aligned} V_B &= E_0^*[\int_0^{\overline{T}} e^{-rt} D(t)dt] \\ &= Q\xi\{[X(0) - \alpha^*]\frac{1 - e^{-(r+\kappa)\overline{T}}}{r + \kappa} + (\alpha^* - G_B)\frac{1 - e^{-r\overline{T}}}{r}\} \end{aligned} \tag{6.21}$$

The right-hand side of the equation consists of two terms. The second term is just the present value of an annuity of $(\alpha^* - G_B)$ which pays from time 0 to time $\overline{T}$, where $(\alpha^* - G_B)$ can be viewed as the long-term risk-adjusted mean spark spread price within one time unit. The second term is discounted with the risk-free interest rate. The first term can be viewed as a correction term. It depends on how far away the initial value of $X(0)$ is from the long-term risk-adjusted mean spark spread price. And the first term is discounted with the risk-adjusted rate, which equals the sum of the risk-free rate, r, and the mean reversion rate, κ. Equation (6.21) implies that not only the long-term equilibrium price but also the current level of spark spread have an impact on a base-load power plant value.

Next we consider a peak-load power plant. We assume the variable cost (excluding fuel cost) of generating one unit of electricity with a peak-load power plant is G_P. Normally, we expect G_P to be greater than G_B. Following the analysis in [BE95], the value of a European call option written on $D(t)$ with a strike price of G_P and a maturity time of T is obtained from

$$V_0[D(T)^+] = Q\xi e^{-rT}\{(\mu^* - G_P)N[d] + \sigma^* n[d]\} \tag{6.22}$$

where

$$d = \frac{\mu^* - G_P}{\sigma^*} \tag{6.23}$$

and μ^*, $\sigma^*$are given by equation (6.13) and (6.14) respectively, $N[\cdot]$ and $n[\cdot]$ are the standard normal cumulative distribution and density functions, respectively.

Note that a peak-load power plant will shut down when the $X(t)$ takes a negative value. Thus the value of a peak-load power plant with a capacity of Q, a capacity factor of ξ, and a life time of $\overline{T}$, can be obtained from integrating the right-hand side of equation (6.22), i.e.,

$$\begin{aligned} V_P &= V_0[\int_0^{\overline{T}} D(t)^+ dt] \\ &= Q\xi \int_0^{\overline{T}} e^{-rt}\{(\mu^* - G_P)N[d] + \sigma^* n[d]\}dt \end{aligned} \tag{6.24}$$

where μ^*, σ^*, d are given by equation (6.13), (6.14) and (6.23), respectively, $N[\cdot]$ and $n[\cdot]$ are the standard normal cumulative distribution and density functions, respectively.

This result is similar in form as the peak-load power plant valuation formula in [NF04a]. The reason for this similarity is that both the two-factor model in [NF04a] and the Ornstein-Uhlenbeck model we used here assume the underlying spark spread prices follow a normal distribution. One major difference is that the spark spread prices in this Ornstein-Uhlenbeck model are "observable" in the markets, while the short-term and long-term equilibrium prices in the two-factor model in [NF04a] are not.

6.3 Spark Spread Series and Model Estimation

For the estimation of the spark spread model parameters, we use the historical daily base-load electricity prices from APX and the TTF (virtual Title

Transfer Facility) gas prices from Endex. The price data range from July 1, 2003 to February 14, 2005. We take Februry15, 2005 as the report date (day 0) for the option valuation analysis in later sections. The APX spot electricity prices in the Netherlands can track back to year 1999, but the TTF gas hub in the Netherlands was only operational since July 2003. Furthermore, the introduction of emission allowance trading in 2005 implies a structural change in the market [2], not only because the CO_2 emission cost will reduce the spark spread values systematically, but also some new embedded options are involved[3]. In order to focus on our option valuation framework, we control for the impact of emission allowance trading by only using the data before the official launch of CO_2 trading in ECX.

In order to construct the spark spread series, we use a constant heat rate of 1.7[4]. The spark spread series is plotted in Figure 1 against electricity prices. We can observe several important properties of the spark spread price curve. Firstly, the spark spread price curve resembles the electricity price curve and shows a strong mean reversion. This is because electricity and gas prices are both mean reverting (see for example in [MT02], [DP94], [AC05] and [KR05]). Secondly, price jumps appear when electricity prices jump to extreme values, but are much rarer and lower in magnitude than in electricity prices. As a result, the spark spreads are less volatile than electricity prices. During the sample period, the annualized volatility of spark spread is 142.61, while the volatility of electricity is 832.76. Thirdly, the seasonal patterns in spark spread are not significant, at least not as significant as in electricity prices. The above three properties of spark spreads can be explained by the positive correlation between electricity and gas prices. Recall that in Chapter 5 we estimate a correlation of 0.4431 and 0.2231 between electricity and gas prices for the winter and summer weekdays, respectively. Electricity and gas prices tend to move in the same direction, thus the volatility, jumps and seasonal differences are offset to some extent. Lastly, spark spread can go negative.

[2]The official trading of carbon dioxide EU allowances (EUA) in the European Climate Exchange (ECX) starts in April 2005.

[3]As we have discussed, emission allowance itself is a real option. According to EU ETS (Emission Trading Scheme), if an operator does not hold sufficient allowances to meet its total emissions at the compliance date, a penalty of €40 (rising to €100 in the second phase) per excess tonne will apply.

[4]Both electricity price and gas prices are in the unit of Euro/MWh, so the heat rate has no unit.

For simplicity of analysis, we assume the invested base-load and peak-load power plants share the same heat rate, although this may rarely be true in reality.

Note that the heat rate of 1.7 refers to a highly efficient power plant. If we choose a higher heat rate (i.e., a less efficient power plant), the possibility of a negative spark spread will be higher. An Ornstein-Uhlenbeck process is a suitable model since it allows for negative spark spread values.

According to the above arguments, we neglect the seasonal pattern in the spark spreads when estimating the model parameters, just as in [NF04a]. Furthermore, we treat all spark spread prices above 300 Euro/MWh as outliers and replace them with 300 Euro/MWh.

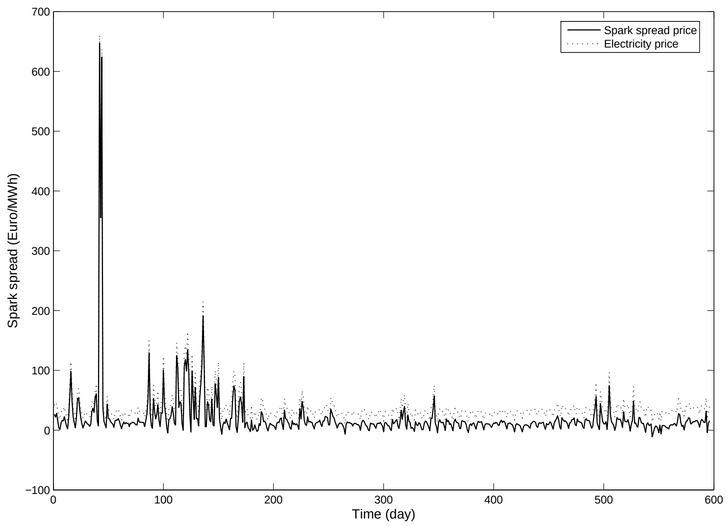

Figure 6.1: Historical spark spread curve and electricity curve

We estimate the mean reverting model following the approach introduced in [DP94]. For simplicity, we assume the market price of risk to be zero. Later in Section 6.7, we will use price information in the forward curve to estimate a market-implied market price of risk. The risk-neutral continuous-time process in equation (6.9) can be discretized into a first-order autoregressive process as

$$x_t - x_{t-1} = \alpha(1 - e^{-\kappa}) + (e^{-\kappa} - 1)x_{t-1} + \epsilon_t \tag{6.25}$$

where ϵ_t is normally distributed with mean zero and standard deviation σ_ϵ, and

$$\sigma_\epsilon = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa}). \tag{6.26}$$

By running the regression

$$x_t - x_{t-1} = a + bx_{t-1} + \epsilon_t \tag{6.27}$$

we can estimate the parameters as

$$\alpha = \frac{-\hat{a}}{\hat{b}} \tag{6.28}$$

$$\kappa = -\ln(1+\hat{b}) \tag{6.29}$$

$$\sigma = \hat{\sigma}_\epsilon \sqrt{\frac{\ln(1+\hat{b})}{(1+\hat{b})^2 - 1}} \tag{6.30}$$

where $\hat{\sigma}_\epsilon$ is the standard error of the regression. The estimated parameters are reported in Table 6.1.

parameter	κ	α	σ
unit		Euro/MWh	
value	0.9133	11.7784	12.9206

Table 6.1: estimated and assigned parameters

6.4 Power Plant Value Sensitivities

In order to compute the value of a power plant, we need to assign some operational configurations of the power plant. We assume the power plant to be invested has a capacity of 400 million megawatt. The lifetime is 25 years. For 85% of the time it can be run to generate electricity. The base- or peak-load power plant has a variable cost of 4.25 or 5.45 Euro for each megawatt of its output, respectively. This variable cost may contain all costs that can be allocated to unit output, such as depreciation, salaries, emission costs, and etc., but not including the fuel cost. The investment cost for a base-load power plant is 250 million Euro, while a peak-load plant costs 300 million Euro. These costs are assumed to increase with the risk-free interest rate 0.06 per year. For a base case scenario, we assume an initial spark spread value of 2.07 Euro/MWh. These assigned parameters are listed in Table 6.2[5].

[5] The power plant configurations are based on [NF04a] and [NF04b].

parameter	Q	X_0		ξ	$\bar{T}$
unit	MW	Euro/MWh			Year
value	400	2.07		0.85	25
parameter	r	G_B	G_P	I_B	I_P
unit		Euro/MWh		Million Euro	
value	0.06	4.25	4.75	250	280

Table 6.2: Power plant configurations

Our purpose in this section is to examine the influences of different parameters on the power plant value. At each time in the following analyses, we alter the parameter of interest and keep all other parameters retain their base case values as given in Table 6.1 and Table 6.2.

First, we look at the effect of the lifetime of the power plant, $\bar{T}$. The curves in Figure 6.2 plot the power plant value as a function of its lifetime. We can see that the power plant value increases with its lifetime, but converges to one value when the lifetime is close to 50 years. The convergence of the power plant value can be explained by the mean reversion of spark spread series. Mean reversion makes the cash flows of the power plant in far future into a perpetual annuity, which has a constant value under a constant interest rate. We can also see that the peak-load plant value is always above the base-load plant value. The value differences between these two types of plants reflect the option type flexibility of a peak-load plant, with which a peak-load plant can choose to run or not to run according to differences between the spark spread price and the variable cost.

As we have pointed out earlier, a base-load power plant resembles a string of forward contracts, while a peak-load power plant resembles a string of option contracts. This is reflected in Figure 6.3, where the power plant values as a function of volatility, σ, are plotted. It can be seen that the peak-load plant value increases significantly with the volatility of spark spread while the base-load plant value is insensitive. With higher volatility, spark spreads are more likely to go below variable costs, and therefore, peak-load plant can avoid more losses in contrast to a base-load plant. If we look back at equation (6.21) and (6.24), we can find that σ does not play a role in the calculation of a base-load power plant value, but does appear in calculating a peak-load plant value.

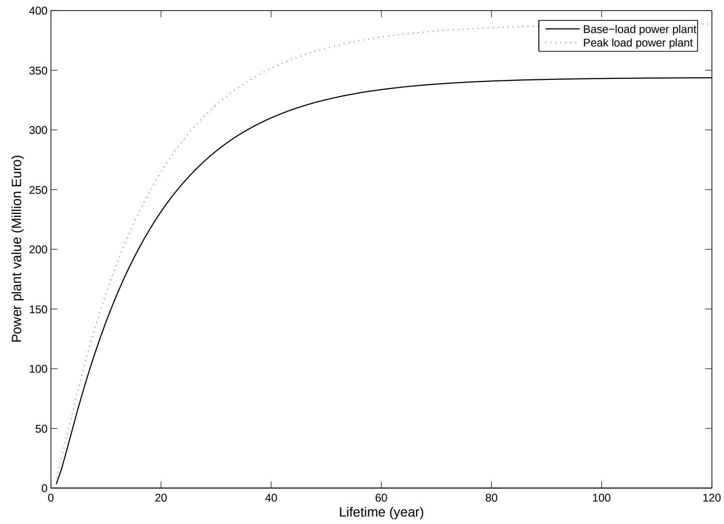

Figure 6.2: Power plant value as a function of lifetime, $\bar{T}$

Figure 6.4 plots the power plant value as a function of the initial spark spread, X_0. We find that the base-load curve is just an increasing straight line, while the peak-load curve resembles the curve of a call option. For sufficiently negative spark spreads, the base-load plant value can become negative, while the peak-load plant value remains always above zero. When the initial spark spread increases the base-load plant value moves closer to the peak-load plant value.

Figure 6.5 plots the power plant value as a function of the equilibrium spark spread, α. We can see that the base-load curve is an increasing straight line, while the peak-load curve resembles the curve of a call option. With sufficiently high equilibrium spark spreads (when $X_0 \geqslant 18.10$ Euro/MWh), a peak-load plant will be very likely to be run around the clock just as a base-load plant. The lower variable cost of a base-load plant will make it more profitable than a peak-load plant. The arguments are similar as those for the initial spark spread, X_0, but the initial spark spread and the long-term equilibrium spark spread are different concepts. Note that X_0 reflects the observed spark spread price in current market situation, it is dynamic; while α reflects the long-term equilibrium spark spread price level, it is structurally exogenous and is assumed constant in our one-factor model.

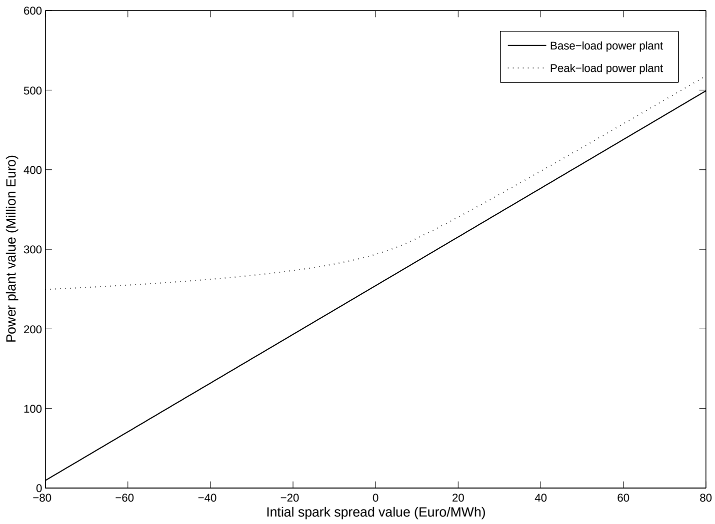

Figure 6.3: Power plant value as a function of spark spread volatility, σ

Next, we analyze the power plant value sensitivity with regard to the mean reverting rate, κ. The results are reported in Figure 6.6. The three panels correspond to three scenarios where the deviations of initial spark spread from spark spread equilibrium value are different. In all scenarios, the peak-load plant value decreases with κ, and converges to a flat line when κ grows greater than 10. The impacts of κ on base-load plant value are more complex and differ among the three scenarios. When $X_0 < \alpha$ (upper panel), the base-load plant value increases with κ, and converges to a flat line eventually. When $X_0 = \alpha$ (middle panel), base-load plant value is insensitive to κ. When $X_0 > \alpha$ (bottom panel), the base-load plant value decreases with κ (the decreasing speed is slower than that of peak-load plant value), and converges to a flat line eventually.

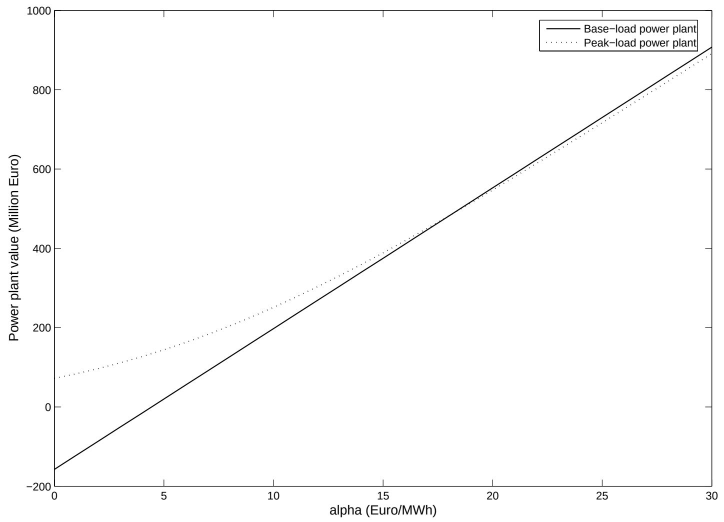

Figure 6.1:

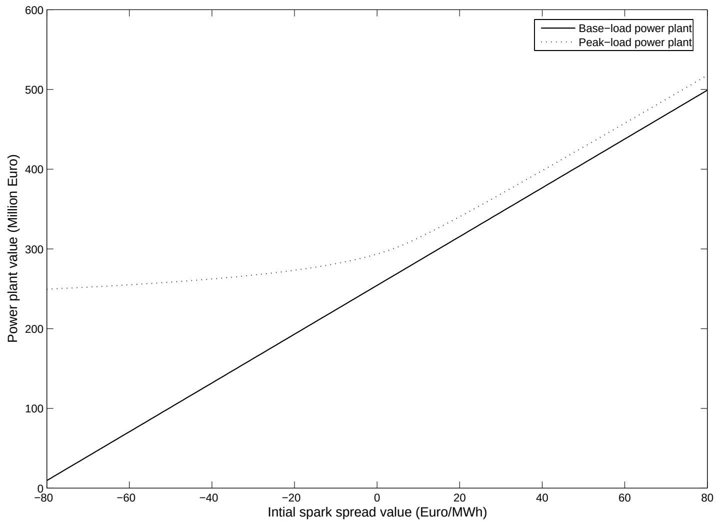

Figure 6.4: Power plant value as a function of initial spark spread value, X_0

Figure 6.5: Power plant value as a function of equilibrium spark spread, α

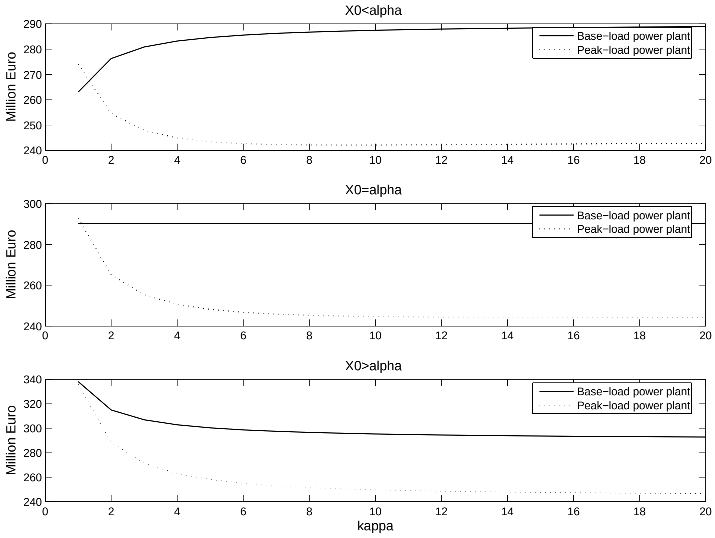

Figure 6.6: Power plant value as a function of mean reversion rate, κ

The impact of κ on a peak-load plant value can be explained by the option type contract for a peak-load plant. Recall that call option values increase with the underlying volatility, as shown in Figure 6.3. In equation (6.14), we find that the spark spread volatility σ is a decreasing function of κ. As κ goes to infinity, σ converges to zero. This explains why the peak-load plant value always decreases with κ. For a base-load power plant, the spark spread volatility σ has no impact on the power plant value, as shown in Figure 6.3. Instead, κ participates in computing a base-load plant value through the expected spark spread $E[X]$, which is given by equation (6.13). Figure 6.7 displays different scenarios for equation (6.13). It is easy to find that the way κ affects the expected spark spread $E[X]$ depends on the initial spark spread level, X_0. When $X_0 < \alpha$, κ is the force pushing the spark spread up to the long-term equilibrium level, α. When $X_0 > \alpha$, κ is the force pulling the spark spread down to α. Note that in Figure 6.7, α keeps on a constant value given in Table 6.1. The different directions of mean reverting force κ lead exactly to the different impacts on the base-load power plant value in Figure 6.6.

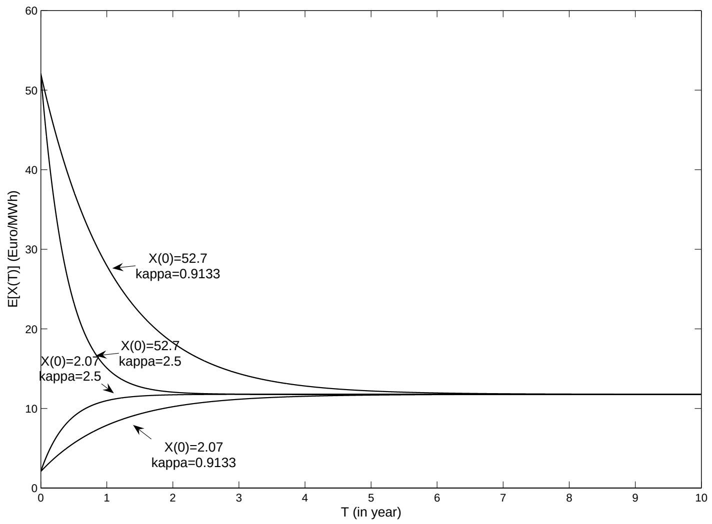

Figure 6.7: Expected spark spread value, $E[X]$

The variable costs, G_B and G_P, and the market price of risk λ, are the terms to be deducted from the spark spread when calculating the cash flows of a power plant. As expected, they both have a negative impact on the power plant value. The impact of variable costs are plotted in Figure 6.8. It can be seen that the base-load plant value decreases linearly with G_B, while the decreasing effect from G_P is partly absorbed by a peak-load plant. The impact of λ has the same shape as that of G_B and G_P, so the curve of λ impact is not reported here. In practice, the information of variable costs can be collected from the operational details of a power plant. However, the market price of risk must be derived from the financial markets. In Section 7, we will conduct an estimation of the market price of risk.

Figure 6.9 plots the changes of power plant values when the risk-free interest rate r changes. With the increase of r, the values of both type of power plants decrease almost in parallel. The curves resemble the curve of an inverse function. This effect is consistent to our expectation since r serves as the discount factor in determining the power plant values.

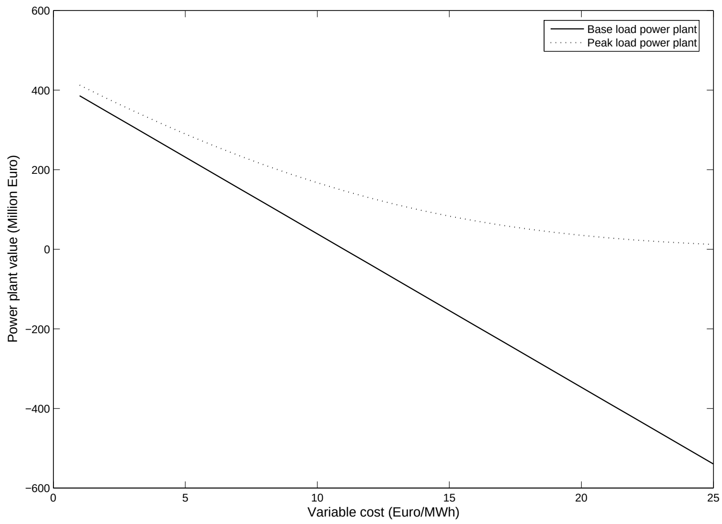

Figure 6.8: Power plant value as a function of variable cost, G_B and G_P

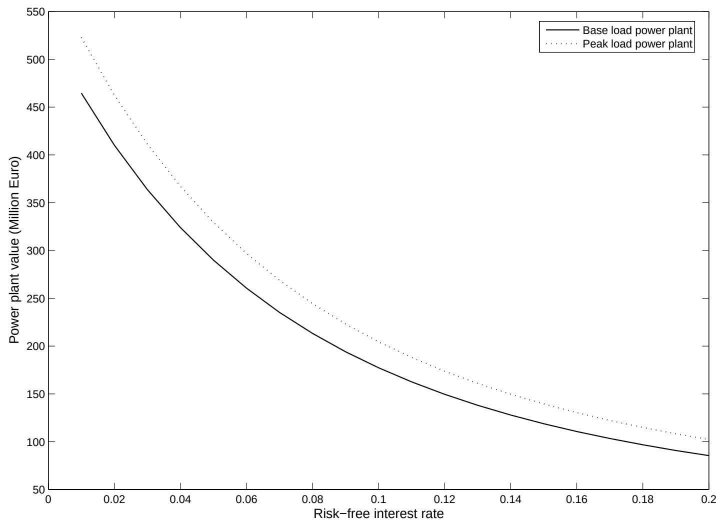

Figure 6.9: Power plant value as a function of risk-free interest rate, r

In previous sensitivity analyses, we have used the same approach: Examine the power plant value changes by letting the parameter of interest change while other parameters remain constant. However, it is more informative to investigate the joint impacts of two different parameters on power plant values. Figure 6.10 is an example of such joint analyses. When the initial spark spread X_0 is low, the base-load power plant value increases with κ; when X_0 is high, the power plant value decreases with κ. These are exactly the results in Figure 6.6. Note again that α keeps on a constant value as given in Table 6.1.

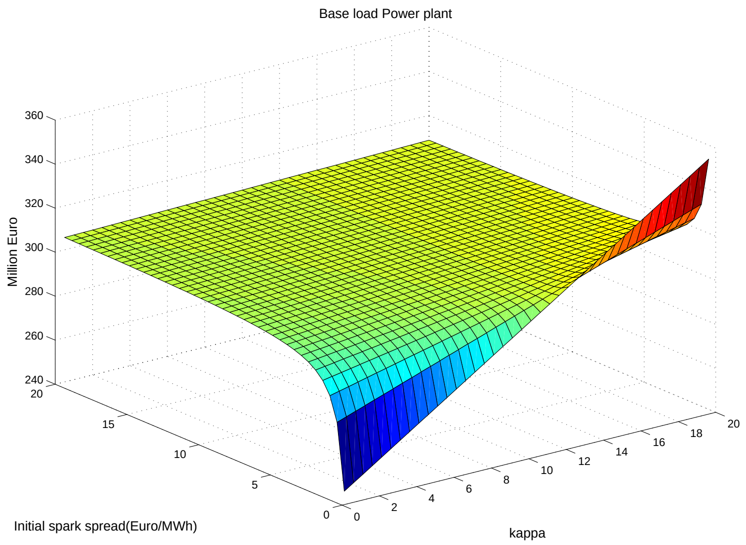

Figure 6.10: Joint effects of X_0 and κ on a base-load plant value

The sensitivity analyses above shed some lights on the role that each parameter plays in determining the value of a power plant. More importantly, we have continuously found the consistency between the analysis results and operational realities of power plants. These findings, from one angle, validate the power plant valuation formulas. In the following sections, we will use these formulas repeatedly for the valuation of power plant investment options. Within our framework, the power plant value will be determined as a function of the observed initial spark spread price, which is the only factor in our one-factor model.

6.5 The Option to Invest in a Base- or Peak-load Power Plant

In this section, we valuate the investment options in a base- or peak-load power plant under various scenarios. The maturity of the investment option varies from zero, infinite time, to a finite time period. The option to invest in an expandable power plant is also considered. In the cases where a closed-form solution is not available, we use both the Hull and White trinomial tree and the Least Squares Monte Carlo method.

6.5.1 Now-or-Never Investment

For a now-or-never opportunity to invest in a base- or peak-load power plant, the option value is simply determined by comparing the invested power plant value and the investment cost. Let F_B, F_P, and F_C be the option value of investing in a base-load, peak-load, base- or peak-load power plant, respectively. The values are given by

$$F_B = \max(V_B - I_B, 0) \tag{6.31}$$

$$F_P = \max(V_B - I_P, 0) \tag{6.32}$$

$$F_C = \max(V_B - I_B, V_P - I_P, 0) \tag{6.33}$$

where I_B and I_P are the investment cost of a base-load and a peak-load power plant, respectively.

With the parameters in Table 6.1 and Table 6.2, we obtain the graphs of now-or-never option values as a function of the initial spark spread value in Figure 6.11. The solid line is the curve for F_B, and the dashed line is the curve for F_P. At each initial spark spread value, F_C takes the higher value of F_B and F_P, so it is the upper envelop of the latter two curves.

In Figure 6.11, F_P has the shape of a call option value, while F_B has the shape of the pay-off function of a call option. This is because both F_B and F_P are options with a maturity of zero, and a base-load plant resembles a forward contract while a peak-load power plant resembles a string of European call options. Since V_B can be calculated easily with equation (6.21), so does F_B. F_P is actually a compound option. Valuation of simple compound options has a closed-form solution, which can derived from the Black-Scholes [Ges79]. Due to the complex payoff function of F_P, a closed-form calculation will be tedious. Alternatively, we use numerical integration to compute V_P with equation (6.24) and calculate F_P with equation (6.32).

Both F_B and F_P are monotonically nondecreasing in X_0. When X_0 increases from an extremely negative value, F_P begins to take a positive value earlier (when $X_0 = -17.34$ Euro/MW) than F_B (when $X_0 = -5.96$ Euro/MW). F_B increases faster than F_P and climbs above F_P at $X_0 = 10.86$ Euro/MWh. At this point, there is no difference between investing in a base-load plant and in a peak-load plant. When $X_0 < 10.86$ Euro/MW, a peak-load plant is preferable than a base-load plant. When $X_0 > 10.86$ Euro/MW, it is better to invest in a base-load plant.

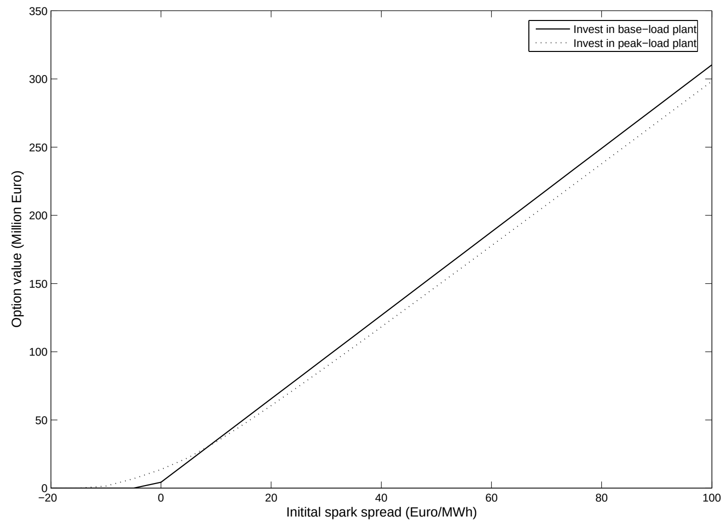

Figure 6.11: Now-or-never investment option values

As we pointed out earlier, the investment right is usually auctioned. Let us assume the government offers a licence to invest either in a base- or peak-load power plant with all the technological details specified in Section 6.4, and the investment must be executed right after the licence is issued. The market price of this licence is then equal to the value of F_{both}. For example, when $X_0 = 2.07$ Euro/MWh, the licence value is $F_{both}(2.07) = F_P(2.07) = 16.98$ million Euro. A peak-load power plant is the optimal choice.

To decide the best bidding price, a bidder must add a premium to the market value of the licence according to its competitive position and strategic considerations. The determination of the optimal premium is beyond the scope of this thesis, and is therefore excluded in our discussion.

6.5.2 Perpetual Option to Invest

Now we consider the case in which the licence holder can wait to invest. Let $F(X)$ denote option value to invest in a power plant, where X is the spark spread price which follows the Ornstein-Uhlenbeck process described in equation (6.9). Although the spot markets of electricity and gas are not complete, all the derivatives payoffs can be replicated by trading futures or forward contracts [DJS01]. In another word, all the uncertainties in the derivatives can be spanned by tradable assets (see for example in [Duf01] and [DP94]). Using the contingent claim method, $F(X)$ must satisfy the following differential equation ($F(X)$ is sometimes written as F for simplicity):

$$\frac{1}{2}\sigma^2 F_{XX} + \kappa(\alpha^* - X)F_X + F_t - rF + D = 0 \tag{6.34}$$

where D is the instantaneous cash flow generated by the project. The investment option does not generate any cash flow during the holding period of the option, so D is equal to zero. And for a perpetual investment option, the term with a partial derivative with regard to time t disappears. Then we have

$$\frac{1}{2}\sigma^2 F_{XX} + \kappa(\alpha^* - X)F_X - rF = 0 \tag{6.35}$$

The equation must satisfy the following boundary conditions:

$$F(-\infty) = 0 \tag{6.36}$$

$$F(X^*) = V(X^*) - I \tag{6.37}$$

$$F'(X^*) = V'(X^*) \tag{6.38}$$

where X^* is the threshold spark spread at which it is optimal to invest.

The first condition above says that as the spark spread takes large negative values, the power plant value goes to zero, so does the investment option value. The second and third condition are just the value-matching and smooth-pasting boundary conditions as introduced in [DP94].

We first consider the perpetual option to invest in a base-load power plant, where the plant value is given by equation (6.21). We rewrite equation (6.21) as

$$V_B = \Phi X + \Theta \tag{6.39}$$

where

$$\begin{aligned}\Phi &= Q\xi\frac{1-e^{-(r+\kappa)\overline{T}}}{r+\kappa}\\ \Theta &= Q\xi[\alpha^*\frac{e^{-(r+\kappa)\overline{T}}-1}{r+\kappa}+(\alpha^*-G_B)\frac{1-e^{-r\overline{T}}}{r}]\end{aligned}$$

The three conditions then become

$$F(-\infty) = 0 \tag{6.40}$$

$$F(X^*) = V(X^*) - I_B \tag{6.41}$$

$$F'(X^*) = \Phi \tag{6.42}$$

The PDE given in equation (6.39) can be transformed into a Kummer's Differential Equation [AC05], to which a general closed-form solution exists. This solution includes a Tricomi's or second-order hypergeometric function and a Kummer's or first-order hypergeometric function [AC05]. Applying the bounding conditions to the general solutions, the coefficients to the Tricomi's and Kummer's functions can be determined. Due to the lengthiness and complexity, we exclude the full derivation in this thesis.

With the parameters given in Table 6.1 and Table 6.2, the optimal spark spread price to invest in the base-load power plant is calculated to be 54.68 Euro/MWh. We can easily prove that at the optimal spark spread value, the base-load power plant value is higher than the investment cost, i.e., $V_B(X^*) > I_B$. This relation justifies the value of the option to wait. Furthermore, the relatively high triggering spark spread price is largely correlated with the perpetual property of the option and the high volatility of the spark spread process.

When comes to the investment option for a peak-load power plant in infinite time, recall equation (6.24) which gives the value of the power plant as a function of the initial spark spread, both the first- and second-order derivatives with regard to the spark spread are not analytically solvable. Thus, equation (6.35) does not have a closed-form solution. Furthermore, for both the base- and peak-load power plant investment options, there is not a closed-form solution when the option maturities are finite, which means the term F_t cannot be eliminated from equation (6.34). In these cases where a closed-form solution is absent, numerical methods are required to solve for the option values. In short, within the framework we used in this chapter, a closed-form solution to the value of the option to invest only exists in the case to invest in a base-load power plant in infinite time.

6.5.3 Option to Invest in a Finite Time Period

In this subsection, we consider the options with a finite maturity. As we already know, these option values do not have a closed-form solution. In stead, we rely on numerical methods, namely the Hull and White Trinomial tree and the Least Squares Monte Carlo method, to valuate these investment options.

Hull and White Trinomial Tree

The Hull and White model is a single-factor short-term interest rate model, in which the state variable is normally distributed and subject to mean reversion. With this model, analytical solutions exist for discount bonds and plain-vanilla options. However, for exotic derivatives with path dependence or more complex payoff functions, numerical methods are necessary. Hull and White introduce a tree-building method which is computationally effective [HW94a] [HW96].

The Hull and White model is a no-arbitrage interest rate model in which the parameters are time varying in order to match the term structure at each time point. Due to the absence of sufficient term structure data, we consider a simplification of Hull and White model. In our analysis, all the parameters are estimated from historical data and are not time varying. At any time, we assume the term structure is exactly the forward curve implied from the model, so as defined in equation (6.12).

We follow the Hull and White tree-building method for our option pricing purpose. The procedure involves dividing the process of equation (6.9) into two processes. The first process has an initial value of zero. This process is given by:

$$dX^*(t) = -\kappa X^*(t)dt + \sigma dZ^*(t) \tag{6.43}$$

The second process is the difference between the risk-neutral spark spread process defined in equation (6.9) and the first process defined by equation (6.43).

$$\varphi(t) = X(t) - X^*(t) \tag{6.44}$$

By equation (6.9) and (6.43) and we have

$$d\varphi(t) = [\kappa\alpha^* - \kappa\varphi(t)]dt \tag{6.45}$$

Equation (6.45) is a separable first order ODE and the solution to $\varphi(t)$ is equal to the forward curve at $t = 0$, which is defined by equation (6.12).

So the second process is simply given by

$$\varphi(t) = e^{-\kappa t}X(0) + (1 - e^{-\kappa t})\alpha^* \tag{6.46}$$

If the option has a maturity of T years and we use N steps, we have the time interval for each step equals to $dt = T/N$. The second process has one certain value at each time t. The diffusion is captured in the first process, where the trinomial trees are built.

The trinomial tree for the first process starts at value zero. At each node (i, j) before maturity, the value of X^* can move to three magnitudes to the next time period $i + 1$ with varying possibilities. In order to minimize the approximation error, the jump size is set to be constant at $\sigma\sqrt{3dt}$. In order to ensure the risk-neutral probabilities at each node to be positive, Hull and White introduce the maximum number of successive upward jumps, J_{max}, which is set to be the smallest integer greater than $0.184/(\kappa dt)$. When J_{max} of upward jumps happen, the price can no longer increase. Instead, the price then can remain, go down by $\sigma\sqrt{3dt}$ or go down by $2\sigma\sqrt{3dt}$. In the opposite direction, J_{min} is set to be $-J_{max}$. Thus, to construct a trinomial tree, we start at node $(0, 0)$, $X^*(0, 0) = 0$. If we are at node (i, j), then, we have three choices for the branches illustrated in Figure 6.12.

- Branch (A) is used if $J_{min} < j < J_{max}$;
- Branch (B) is used if $j = J_{min}$;
- Branch (C) is used if $j = J_{max}$.

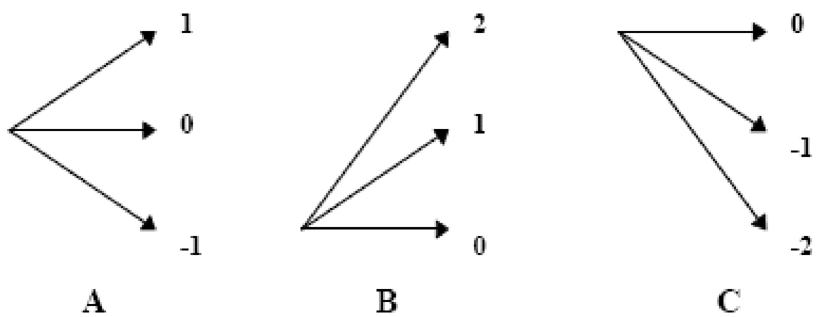

Figure 6.12: Hull and White tree branches

The constructed trinomial tree for X^* is symmetric about the horizontal line $j = 0$, and J_{max} is the number of nodes on either side of the line $j = 0$. At each node (i, j), let P_u, P_m, P_d denote the risk-neutral probability of moving to the highest, middle, and lowest node of time $(i+1)dt$, respectively. Then we have

$$X^*(t, j) = j\sigma\sqrt{3dt} \tag{6.47}$$

It turns out that at (i, j), P_u, P_m, P_d only depends on j. Constraining on the mean and variance of $X^*(i, j+1)$, $X^*(i, j)$, $X^*(i, j-1)$ and using $P_u + P_m + P_d = 1$, we can solve for the probabilities for each type of branches. For branch (A)

$$P_u = \frac{1}{6} + \frac{(j\kappa dt)^2 - j\kappa dt}{2} \tag{6.48}$$

$$P_m = \frac{2}{3} - (j\kappa dt)^2 \tag{6.49}$$

$$P_d = \frac{1}{6} + \frac{(j\kappa dt)^2 + j\kappa dt}{2} \tag{6.50}$$

For branch (B)

$$P_u = \frac{1}{6} + \frac{(j\kappa dt)^2 + j\kappa dt}{2} \tag{6.51}$$

$$P_m = -\frac{1}{3} - (j\kappa dt)^2 - 2j\kappa dt \tag{6.52}$$

$$P_d = \frac{7}{6} + \frac{(j\kappa dt)^2 + 3j\kappa dt}{2} \tag{6.53}$$

For branch (C)

$$P_u = \frac{7}{6} + \frac{(j\kappa dt)^2 - 3j\kappa dt}{2} \tag{6.54}$$

$$P_m = -\frac{1}{3} - (j\kappa dt)^2 + 2j\kappa dt \tag{6.55}$$

$$P_d = \frac{1}{6} + \frac{(j\kappa dt)^2 - j\kappa dt}{2} \tag{6.56}$$

The final tree is constructed by adding the second process, i.e., the forward prices to the trinomial tree for X^*. An illustration of the final tree is graphed in Figure 6.13. In a Hull and White trinomial tree, the price changes in the underlying are additive, which is capable of handling a process that allows negative values. The popular CRR tree [CRR79] used in the Black-Scholes framework does not work when the state variable follows an Ornstein-Uhlenbeck process. Furthermore, the Hull and White tree has the advantages of minimizing the approximation error in the mean and variance, and mimicking the mean reversion in the underlying.

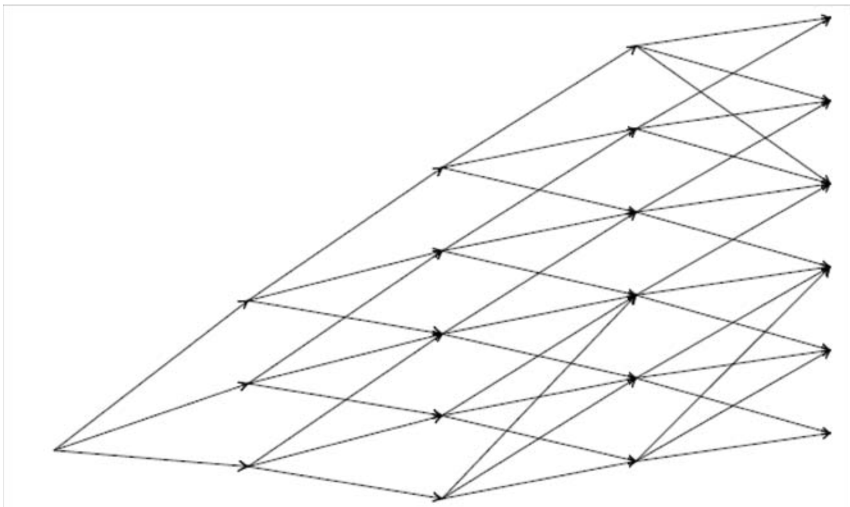

Figure 6.13: Illustration of a complete Hull and White tree

Our initial purpose is to value the investment option at $t = 0$. The backward induction procedure can then be used in the same way as in a CRR tree when valuing an option. Take the base-load investment option for example, the payoff of the j-th path at maturity is

$$F(I_{max}, j) = \max[V_B(X((I_{max}, j) - e^{rI_{max}dt} I_B, 0] \qquad (6.57)$$

At each node before maturity, the option value is equal to the greater of the continuation value, i.e., the value of waiting until the next period, and the exercising value, i.e., the value of investing in the power plant right now. Note that in a Hull and White tree, we need to distinguish three types of nodes which are determined by the three types of branches they have for the next time step. For nodes that are neither at the upper edge nor at the bottom of the tree, type A branch applies. The option values are given by

$$\begin{aligned} F(i,j) &= \max\{V_B(X((i,j) - e^{ridt} I_B, \\ &\quad e^{-rdt}[P_u F(i+1, j+1) + P_m F(i+1, j) \\ &\quad +P_d F(i+1, j-1)]\} \end{aligned} \tag{6.58}$$

For nodes that are at the bottom edge of the tree, type B branch applies. The option values are given by

$$\begin{aligned} F(i,j) &= \max\{V_B(X((i,j) - e^{ridt} I_B, \\ &\quad e^{-rdt}[P_u F(i+1, j) + P_m F(i+1, j+1) \\ &\quad +P_d F(i+1, j+2)]\} \end{aligned} \tag{6.59}$$

For nodes that are at the upper edge of the tree, type C branch applies. The option values are given by

$$\begin{aligned} F(i,j) &= \max\{V_B(X((i,j) - e^{ridt} I_B, \\ &\quad e^{-rdt}[P_u F(i+1, j) + P_m F(i+1, j-1) \\ &\quad +P_d F(i+1, j-2)]\} \end{aligned} \tag{6.60}$$

We perform the backward induction iteratively at all previous time steps up to time zero. $F(0,0)$ is just the option value to invest in a base-load power plant. We use the Matlab program to deal with the calculations.

In Figure 6.14, we plot the option values for both base- and peak-load power plants with a maturity of 5 years. The total time step N is set to be 520[6]. Both the the base- and peak-load plant investment option value have the shape of a call option. At each spark spread value, the option value is higher than in Figure 6.11. For example, at $X_0 = 2.07$, the option value to invest in the peak-load power plant is 27.16 million Euro. This is due to the value to wait in the 5 years. We also find that a peak-load power plant investment option has a higher value than that of a base-load power plant investment option when the spark spread is lower than 5.42 Euro/MWh. This threshold to switch from investing in a peak-load to a base-load power plant is higher than the one we found in the now-or-never investment case, which implies again that in a longer time period to make an investment choice, the option to wait is valuable.

[6]In our experiments, when $N > 509$, the option value change is less than 0.1%. In other word, the option value converges.

Least Squares Monte Carlo

The Least Squares Monte Carlo (LSMC) method involves generating a large number of paths for the underlying spark spread process, and regressing on the basis functions with the in-the-money paths at each step [LS01]. Paths of the spark spread are given as an example in Figure 6.15. The mean reversion property of the price process can be observed in the paths. Based on such spark spread paths, the payoff of the option at each time step on each path can be determined. Taking a base-load plant investment as example, at the maturity of the option, the investment value of each path is given by

$$\max[V_B(X_T) - e^{rT} I_B, 0] \tag{6.61}$$

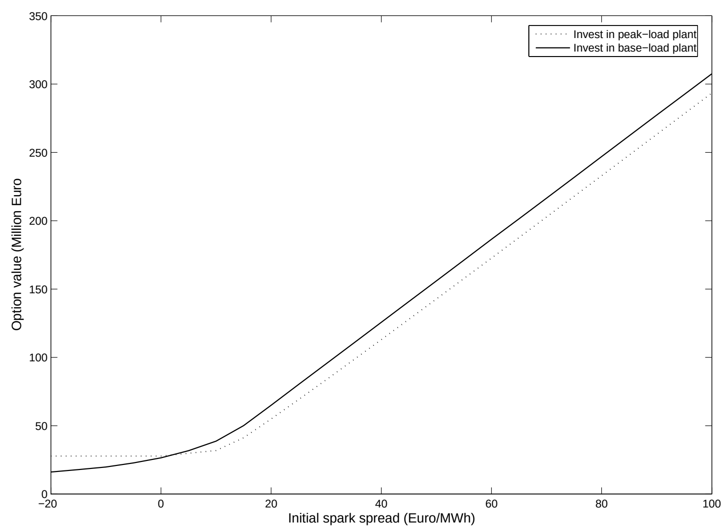

Figure 6.14: Investment option value with a maturity of 5 years

For all the steps before maturity, the continuation value is assumed to be determined by a linear combination of basic functions. Following [LS01], we use the powers of the state variable X as the basic functions. At time step i, we have

$$E_i^Q[F_{i+1}] \approx a_0 + a_1 X_i + a_2 X_i^2 + a_3 X_i^3 \tag{6.62}$$

where E_i^Q is a risk-neutral expectation operator at time i, F_{i+1} is the continuation value of time i, and a_1, a_2, a_3, a_4 are the linear coefficients.

Following [LS01], we only consider the in-the-money paths for the regressions. The parameters at each time step are estimated by ordinary least square method, and then we calculate the continuation value for each path at that time step. The optimal exercise rule is thus to exercise if the continuation value is lower than the exercise value $\max[V_B(X_i) - e^{ri\Delta t} I_b, 0]$ and verse visa. Now we have for all paths the best moments to exercise. For each path, we perform backward induction iteratively up to time zero. The time zero value of the option, $F_{i,0}$, is obtained as the value of the option for that path. Let the total number of paths to be M. The average of the time zero values of all the M paths is just the value of the investment option. That is:

$$F_0 = \frac{1}{M} \sum_{i=1}^{M} F_{i,0} \tag{6.63}$$

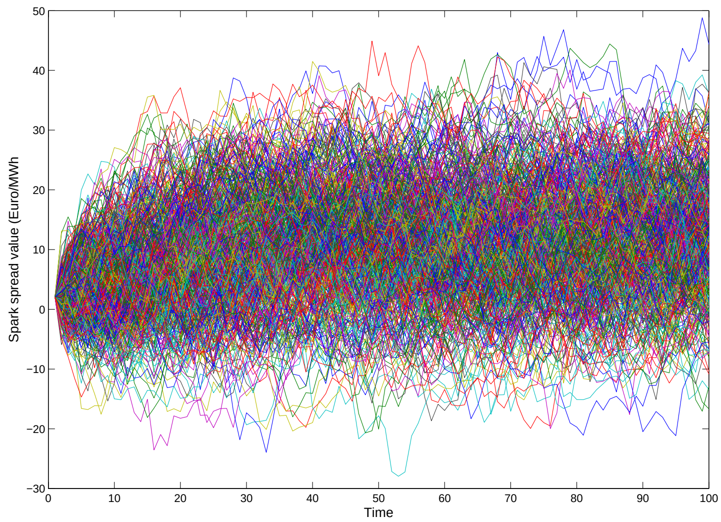

Figure 6.15: Paths of spark spread prices

For the LSMC method, we set the total number of paths M is set to be 25000, and the time step to be one day, i.e., $i = 1/365$[7]. We find the results

[7]With the time step of one day, we are assuming a peak-load power plant can make its decision every day.

are very close to those of the Hull and White tree method in the previous subsection. The average error in the option values between these methods (based only on the integer spark spread values from -20 to 100 Euro/MWh) is 0.0215 million Euro.

It is well known that the LSMC method is subject to computational burden. The computational effort is linear in the number of the number of steps and nearly linear in the number of simulation paths [AB04]. In our experiments, the calculation of the value of a peak-load power plant investment option with a Pentium 4 2.60 GHz computer takes more than 11 hours.

6.6 Option to Expand

In this section, we consider additional flexibility in the opportunities to invest in a power plant. In addition to the option to wait, we may have an option to expand or an option to upgrade the power plant and an option to abandon the project. The option to abandon is proved to be of little value in [NF04b]. In contrast, the option to upgrade from a base-load plant to peak-load plant is much valuable. In [AC06], an option to double the capacity of a base-load power plant is valued with the LSMC method.

Following [AC06], we consider the investment on an expandable power plant. We contribute by analyzing the value differences between investing in a base- and a peak-load plant.

We assume the investment opportunity is a 400 MW power plant that must be built up right now[8] and can be expanded into a 600 MW power plant 5 years later. The expansion can be executed within 5 years. The initial investment cost for such expandable base-load power plant is 275 million Euro. The cost to invest in an expandable peak-load plant is 330 million Euro. The cost to expand the base-load plant is 100 million Euro, and the cost to expand the peak-load plant is 125 million Euro. Our purpose is to value the opportunity to invest in such an expandable power plant. In another word, what is the market price of a licence of investing in such an expandable power plant?

In our experiments, when $M > 22458$, the option value change is less than 0.1%. In other word, the option value converges.

[8] For simplicity, we are assuming here the building time of the power plant to be zero. In [AC06], the building time is taken into account.

As in previous sections, we assume that the underlying variable, the spark spread $X(t)$, follows the stochastic process given by equation (6.1) and the risk-neutral process is given by equation (6.9). We construct the Hull and White trinomial tree to approximate the mean reverting process and obtain the option value by iterative backward induction. We use also the Least Squares Monte Carlo method to value the options to invest in expandable power plants. Again, the results from these two methods are very close. The option values are plotted in Figure 6.16.

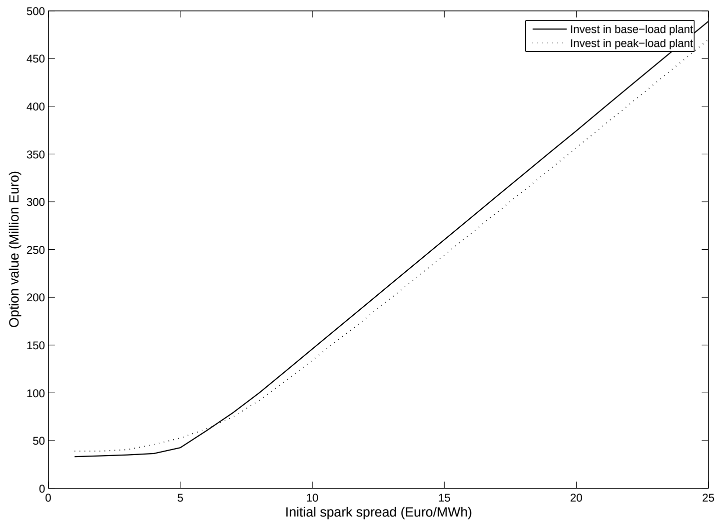

Figure 6.16: Option values in investing in an expandable power plant

The option value curves in Figure 6.16 resemble the shape of the curves in Figure 6.14. However, the curves in Figure 6.16 can be roughly be regarded as the sum of the option value in Figure 6.11 and half of the option value in Figure 6.14, but at different exercise prices. This is because we can decompose the option value into two parts: the option to invest in a nonexpandable power plant, and the option to expand the capacity.

If we assume the investment cost of a power plant is proportional to its capacity, the cost of investing in a 600 MW base-load power plant is 525 million Euro. With an initial spark spread of 2.07 Euro/MWh, the option to invest in a 600 MW base-load power plant is 19.48 million Euro. At the same initial spark spread price, the value of the option to invest in an expandable base-load power plant (first 400 MW, and then expands to 600 MW) is 38.52 million Euro. The value difference, which equals to 19.05 million Euro, is the value of the option to expand.

In the case of investing in an expandable power plant, note that the exercise of the first option is the condition for activating the second option. This makes the first stage of the investment more likely to be executed.

In comparison with the result in Figure 6.11, the initial spark spread threshold to switch from a peak-load power plant to a base-load power plant is lowered to 7.52 Euro/MWh. Recall that the threshold in Figure 6.11 is 10.86 Euro/MWh. When the initial spark spread is low, it is expected to increase in the future, which favors an expand option to invest in a peak-load power plant. In our numerical example, the cost savings of a base- and peak-load expandable power plant are kept at the same scale. Thus, the cost savings difference does not contribute to the change of the switching threshold.

6.7 Market Price of Risk

Standard arbitrage arguments establish the risk-neutral measure for derivatives pricing (see for example in [CIR85] and [LS02]). If we know the market price of risk, we would know the dynamics of the stochastic component of the underlying in the risk-free world and, hence, we could price any derivatives written on the underlying.

In [AC05], the risk-neutral valuation of power plants is performed in the Spanish market. The market price of risk is assumed to be zero simply because there is no traded electricity forward or futures contracts in Spanish power exchange at the reporting date. However, the risk premium in gas prices was estimated with the NYMEX natural gas futures data. Apparently, this asymmetric treatment would lead to a bias in the risk-neutral spark spread process.

The historical spot price data contain no information for the market price of risk. Thus, market price of risk can only be extracted from derivatives markets. One way to obtain the market price of risk is to imply it from option prices [Wer05]. This technique resembles the recovery of the implied volatility with the Black-Scholes model. However the spark spread options are not traded in the market. Neither can the option prices be easily constructed from traded contracts. Alternatively, we turn to imply the market price of risk for the spark spread from the forward curve. Although the forward or futures spark spreads are not traded directly in the markets, we can construct a forward curve under a certain heat rate by using the similar approach for the construction of the spark spread spot price series.

We need both the electricity forward price and the gas forward price to calculate a forward spark spread price. The Dutch electricity and gas futures market Endex only started to publish reference prices for the Dutch TTF gas from February 9, 2005[9]. At the reporting date, February 15, 2005, we have only a few days of observations of the gas futures prices and they are not sufficient to build a meaningful spark spread futures series. This is the reason why we use the historical spot spark spread series rather than the futures spark spread series as the underlying variable.

On the reporting day, February 15, 2005, we observe in Endex in total 8 gas futures contracts. Their maturities are: 1-3 months, 1-2 quarters, 1-2 seasons and 1 year. For each of these delivery period, the electricity futures price can be observed or implied from the traded contracts as well[10]. Thus, on the reporting day, we can construct the "observed" spark spread forward curve. With a fixed heat rate of 1.7, this forward curve is plotted by the eight solid flat lines in Figure 6.17.

Following the method in [CS03], the market price of risk for the spark spread is obtained by minimizing the sum of the squared errors between the "observed" futures spark spread prices and the model implied futures prices, i.e.,

$$\lambda = \arg\min_{\lambda} \sum_{i=1}^{8} (F_i - \hat{F}_i)^2 \tag{6.64}$$

where F_i is the i-th futures contract price "observed" in the market and $\hat{F}_i$ denotes the i-th futures contract price based on the one-factor model in the previous sections. Recall that the model implied term structure is given by equation (6.12). All parameters except λ are known from the estimations in Section 6.3. The model implied price of the i-th futures contract which

[9]The official lauching of trading of TTF gas forward contracts on Endex was on 31 March 2006.

[10]The longest maturity of electricity futures in Endex is 3 calendar years.

requires delivery from time t_1 to t_2 is therefore given by

$$\hat{F}_i = \frac{1}{t_2 - t_1} \sum_{t=t_1}^{t_2} e^{-kt} X(0) + (1 - e^t)\alpha^* \tag{6.65}$$

where

$$\alpha^* = \alpha - \frac{\sigma\lambda}{\kappa}. \tag{6.66}$$

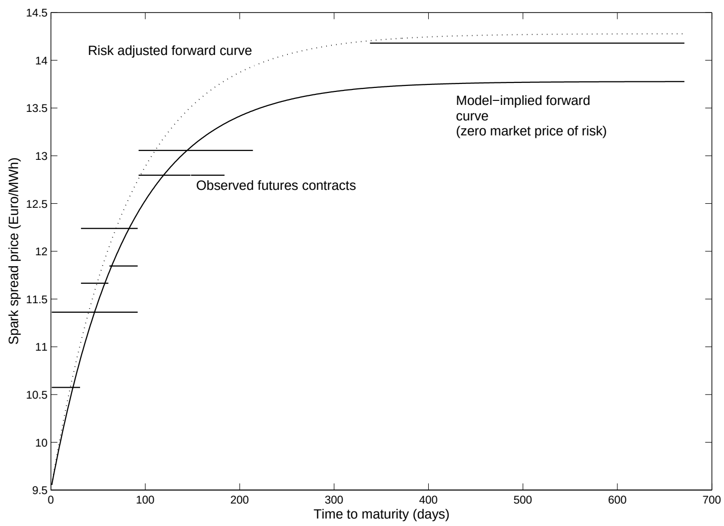

Figure 6.17: Observed, model implied, and risk-adjusted term structures of spark spread on February 15, 2005

By running an Fminbnd function in Matlab, we obtain the implied market price of risk for spark spread to be $\lambda = -0.0354$. Substituting the value of λ back to equation (6.16), we obtain the risk-adjusted term structure which is represented by the dotted curve in Figure 6.18. The solid curve is the model implied term structure assuming a zero market price of risk. At each time of maturity, the vertical distance between these two curves reflects the risk premium that applies in calculating a risk-adjusted cash flow.

A negative market price of risk is widely identified in the energy markets. [Sch97] reports a negative market price of risk for oil with a one-factor model.

[Ron02] finds negative market prices of risk at the Pennsylvania-New Jersey-Maryland (PJM) electricity market. [CF05] reports a negative market price of risk in the England and Wales electricity market. The negative premium is related to compensating the possibility of hourly or daily spikes that exists in the electricity prices and the spark spread prices. Forward or futures sellers who give up the spikiness of hourly or daily prices opportunities by selling at a fixed price must ask for a premium. However, the market price of risk of an energy product is not always negative. In the theoretical discussion in [BCK06], the market price risk premium is related to behavior of the market players. It is argued that if the producers have more market power[11] than the consumers, the market price of risk will be negative to reflect the advantages of the producers. This reflects the properties of a sellers market. In the Dutch electricity market, the producers are believed to have remarkably high market power [NVV03].

Next, let us see how a negative market price of risk would affect the value of a power plant and of an investment option. According to equation (6.16), a negative market price of risk implies an increase in the equilibrium spark spread in the risk-neutral world, and subsequently an increase in the power plant value and investment option values. With all model parameters except λ remain as in Table 6.1 and Table 6.2, the equilibrium spark spread will increase by $-\frac{\sigma\lambda}{\kappa} = 0.5008$. The base-load power plant value will increase by 1.5325 million Euro, and the peak-load power plant value will increase by 0.8678 million Euro. The option to invest in a nonexpandable base-load power plant value in 5 years will increase by 0.4837 million Euro, and the option to invest in a nonexpandable peak-load power plant in 5 years will increases by 0.2433 million Euro.

6.8 Conclusions and Further Research

This chapter provides a framework of valuating power plants and investment licences. The analyses are based on a one-factor mean reverting model for the spark spread process.

In the electricity supply industry, the spark spread is the core value driver of gas-fired generation assets. As a single time series, the spark spread shows

[11] Market power is typically defined as the ability to profitably alter prices away from competitive levels. The European Union defines Significant Market Power (SMP, specifically, in communications markets) as equivalent to the concept of dominance.

several merits. We can avoid the estimation of the correlations between the electricity and the gas prices without losing the correlations information. We can neglect the seasonality pattern in the spark spread series since the positive correlation between electricity and gas prices tends to offset much of their cyclical movements. We show that the Ornstein-Uhlenbeck process is an appropriate description of the spot spark spread series. The closed-form valuation formulas for a base- and peak-load power plant show consistent properties with the operational characteristics of each type of power plants.

The investment licence for power plants is valued as American options. For different types of power plants, we value the investment option by using both the Hull and White trinomial tree and the Least Squares Monte Carlo method. These two methods yield similar results. For all investment options, we find out that the option to invest in a peak-load power plant is worth more than the option to invest in a base-load power plant when the spark spread is lower, and verse visa. The threshold to switching from investing in a peak-load power plant to a base-load power plant increases due to the option to wait.

We estimate the market price of risk from the constructed spark spread forward curve on the report day and find a negative market price of risk for the spark spread in the Dutch markets. This implies more value for the invested power plants and the investment options.

The research along this line can be extended in several directions. Firstly, it is easy to extend the one-factor model to a multi-factor model by letting the equilibrium spark spread to be stochastic. We expect the two-factor model will be more flexible to capture the dynamic term structures of the spark spread, especially when a long series of forward curves is available in the markets.

Secondly, we can take into account the jumps and the seasonal effects of spark spreads in the model. Just as the electricity price distributions, the spark spread prices have as well a fat tail skewed to the right. We expect a richer model will improve the goodness-of-fit of the model, especially in terms of higher order moments. In consequence, the power plant and investment option values can be obtained either by Monte Carlo simulations or with the decomposition approach introduced in [DH03].

Thirdly, we may introduce game theoretic models to reflect the competition between market players. We believe the energy prices will change in response to a new capacity and the first mover advantages do exist.

Lastly and most importantly, we can apply our model to possible real cases where a power plant licence is auctioned or a transaction of existing power plants is made. Comparing the model implied price and the real transaction price provides an efficient way to validate and improve our model.

In the next chapter, we will summarize the main finding in this thesis and present discussions on several relevant issues.

Chapter 7

Concluding Remarks

7.1 Summary of Conclusions

This thesis focuses on the real options applications in the energy markets, especially in the electricity supply industry. The real options method has been used for the valuation of generation assets and the decision-makings in operation and investment. This section provides a summary of the main findings throughout the thesis.

7.1.1 Electricity Prices Modeling

To carry out any real options analysis on a power-related asset or an investment opportunity, the electricity price modeling is the starting point. Electricity prices are notoriously hard to model due to their exotic behaviors. The extraordinarily high volatilities, strong mean reversion, salient cyclical price patterns, and occasional occurrence of price spikes may demand very complicated models.

In Chapter 4, we carry out an empirical analysis with the Dutch and German electricity market data, by using a jump diffusion model and various regime switching models. We find that the regime switching models have a superior performance than the jump diffusion model in terms of replicating the higher moments in the historical price data and forecasting electricity prices. The parameters of a regime switching model are more difficult to estimate than the jump diffusion model, because the former involves an Expectation Maximization method. The derivatives pricing under a regime switching model is less addressed in literature than jump diffusion models.

However, this is not a problem when we use the Monte Carlo simulation method for derivatives pricing.

7.1.2 Power Plant Valuation

The real options method can capture the managerial flexibility in an asset or a project. Thus, the real options valuation is able to reveal more value than the traditional discounted cash flow method. This argument is established in Chapter 2 and Chapter 3. For a peak-load power plant, the operator can shut down the turbines when the spark spread is lower than the variable cost. In a real options framework, a peak-load power plant is modeled as a string of call options on the spark spreads. Nevertheless, the option-based value of a peak-load power plant can be considerably dampened by the volumetric risk factors from both the supply side and the demand side. The supply-side risk is determined by the operational constraints of a power plant. In a real options term, the operational constraints lead to either an increase in the strike price or a decrease in the total volume of the embedded options.

In Chapter 5, we perform the Monte Carlo simulations with and without these operational constraints in a near-reality case. As expected, the power plant values are decreased by each of these constraints. The decreasing effects of some constraints, namely, the forced outage rate, the maintenance rate and the spinning reserve rate, are significant.

Power plants are often obligated to serve an aggregate customer load. When we take the demand-side uncertainty into account, the power plant can be valued as a portfolio, which consists of a string of call options on spark spreads and an aggregate customer load contract. In an efficient spot market, a load-servicing power plant would sell its surplus electricity and buy spot electricity if it is in shot of capacity to meet its customer need. The customer load risk can remarkably deteriorate the profitability of a power plant. The close co-movement of the customer load and the electricity prices may produce disastrous results. For example, when the spot price rockets, the demand also rises high, the power plant has to buy extra power from the spot market at high prices. In Chapter 5, we perform Monte Carlo simulations with a deterministic or a stochastic customer load when price in a forward customer contract is fixed. Even if the predetermined prices in the forward contract is higher than the average of the predicted forward prices, the power plant can not avoid a loss of value from the load uncertainty. A load-servicing power plant must seek other tools to hedge the demand-side risk.

7.1.3 Investment Decision

The spark spread reflects explicitly the profitability of the generation asset in unit time. Although we can not observe the spark spread directly in the markets, we can construct a spark spread series with the traded electricity and gas price data. For power plant valuation and investment analysis, taking the spark spread as the underlying variable has the following advantages. Firstly, we can avoid the modeling of the correlation between the electricity and the gas prices. This correlation is time-varying and burdensome to handle. In a spark spread series, the correlation between the electricity and the gas price is built-in always taken into account. The second advantage of a spark spread series lies in its less significant seasonal patterns than electricity prices. This is due to the positive correlation between the electricity and the gas prices. The co-movements of these two prices tend to offset the seasonalities to some extent. For the same reason, a spark spread series has a lower volatility and less prices jumps than electricity prices. With the dutch market data, our empirical analysis in Chapter 6 supports these conclusions.

The one-factor Ornstein-Uhlenbeck model is a good approximation for the historical spark spread data. The Ornstein-Uhlenbeck model can capture two important properties of a spark spread series: the mean reversion and allowance for negative values. Furthermore, the Ornstein-Uhlenbeck model is tractable and a closed-form solution for the term structure prices is available. This provides much convenience when we perform the Monte Carlo simulations for the cash flows of a power plant with forward prices.

The licence to build a new power plant is usually auctioned in deregulated electricity markets. In Chapter 6, we model the licence as an American option on an operating power plant, with investment cost being the strike price. Moreover, the value of a base-load power plant is the sum of the values of a string of forward contracts on the spark spreads during the lifetime of the power plant, whereas the value of a peak-load power plant is the sum of the values of string of call options on the spark spreads. With the one-factor model, the option to wait contributes great value to the investment opportunity, so does the option to expand the power plant capacity. In all these investment options, a base-load power plant is preferred when the initial spark spread is high, and a peak-load power plant is a better choice when the initial spark spread is low.

7.2 Discussions

In this section, we carry out discussions on four important issues related to our research topic. Studies on these issues can be either a robustness test of the conclusions obtained in this thesis, or an extension to the current research.

7.2.1 Market Efficiency

The deregulation of the electricity markets is recent and the trading of main commodities has only a short history. In continental Europe, the power trading was first introduced in 1999, the Dutch TTF gas hub began to operate in 2003, and the emission allowance trading started in 2005. In the Dutch APX markets, the day-ahead trading volume in 2006 amounts to 19 TWh, but this represents less than 20% of the total electricity consumption in the Netherlands. The wholesale market has not enough liquidity[1]. The market power is believed to exist to some extent, which may be exploited by some dominant players to manipulate the energy prices [NVV03]. In one word, the electricity and gas markets are immature. This fact contradicts the assumptions we have made in our research, where we believe the quoted prices reflect all the information in the markets and all investors are price-takers.

The immaturity of the market yields different impacts on different types of market players. In an immature market, if one investor does not have a dominant power to affect the market, he or she will ask for an extra risk premium for compensation. As a consequence, both the option-based peak-load power plant value and the investment option value will be lower than we have shown in Chapter 5 and Chapter 6. The optimal time to invest in a power plant will be delayed accordingly. In contrast, for a dominant player who has a certain level of market power, the adjustments needed to be done will be exactly in the opposite direction [BCK06].

7.2.2 Forward Price Dynamics

In this thesis, we focus on the spot prices of electricity and gas. This is largely due to the fact that the spot market has a longer history and it is more liquid.

[1]Nord Pool is the most liquid electricity market, where the trading volume in 2006 amounts to about 60% of the consumption.

Electricity is a non-storable commodity, so its expected return is not usually equivalent to the risk-free rate under a martingale measure [NF04a] [NF04b]. For the pricing of the derivatives based on the spot price models, we need to adjust the model by deducting a risk premium from the drift term. In Chapter 6, we use the forward curve on the reporting day to derive the market price of risk for the risk-neutral adjustment.

Electricity and gas forward prices are financial assets, and thus their expected returns under a martingale measure are equal to the risk-free interest rate. In another word, if we believe the forward markets are efficient (again this assumption is questionable, and long-maturity forward prices are less reliable), we could use the forward price as the risk-neutral forecast prices for the future spot prices [NF04].

To date, the Dutch derivatives market Endex has considerable trading records in the forward contracts. The evolution of the forward curves definitely contains important information which reflects the expectation of the markets. For electricity, the current spot price does not necessarily have anything to do with the spot price at some future time point [Wer05]. Similarly, no explicit connections exist between forward prices of different maturities. However, forward prices are conditional expectations, thus the expectations can be affected by the spot prices, especially for the near future prices. That means, if we can filter out the seasonal patterns and structural changes in the market, the forward prices could be handled by the term structure models. Such examples can be found in [CS00], [MT02] and [KO05].

With electricity and gas forward prices, we can construct the forward spark spread prices and estimate the term structure model. An important issue is to remove the seasonal factors in both the forward and the spot prices. According to [KO05], at least two factors are needed for the term structure of spark spread.

7.2.3 Emission Cost

The emission cost has been an important component in determining the profitability of power plants since the European Union Emission Trading Scheme (ETS) was introduced. Phase one of this scheme has realized a mixed result. The cap and trading has been proved to be a successful mechanism for green house emissions reduction. The unsuccessful part of the scheme is that the authorities have allocated too many emission allowances for free. Abundance of allowances pulled the CO_2 quotes prices from above 30 Euro/ton to below

10 Euro/ton.

Chapter 5 assumes a constant CO_2 cost and shows its negative impact on a power plant value. With the CO_2 price data, we can build a stochastic model and add CO_2 price as an additional factor to the model. This brings not much difficulty if we use the Monte Carlo simulation method to value a power generation asset. One important issue is to estimate the correlation between electricity (or spark spread) and CO_2 prices.

Chapter 6 avoids the CO_2 impact by restricting the data sample to the time without CO_2 trading. If we assume the forward markets are efficient, we could expect the forward electricity prices has already reflected the expected CO_2 costs. The increased variable cost from CO_2 will be passed on to the end consumers. Again, we are left to judge how efficient the markets are.

In the year 2005 and 2006, electricity prices increased remarkably. The increases in electricity prices cannot be explained fully by CO_2 costs. Other reasons may include the increased demand, the increased fuel price, the changed climate, and the generators' strategies.

The ETS Phase Two is to start in 2008 in the European Union. Uncertainties about the cap and the national allocation plans (NAPs) still remain. Aggressive emission reduction plans from the UK encourages the EU's persistence to its Kyoto Protocol plan. In 2007, the EU leaders set a firm target of cutting 20% of the EU's greenhouse gas emissions by 2020[2]. Uncertainties in CO_2 costs implies difficulties in power plant valuation. For the investor who holds now a licence to build a new power plant, the best strategy may be to wait for more information about CO_2 prices to come.

7.2.4 Competition and Game Theory

In this thesis, we did not analyze the impact of competition explicitly. On the one hand, we assume that the entry of a new generation capacity does not have any substantial impact on the energy market prices..This assumption is actually based on the market efficiency assumption. In efficient electricity markets, the impact of new generation capacities will be fully reflected in the forward prices. All market participants are price-takers. On the other hand, we assume that the power plant investor has the monopoly on the option to invest. The investment decision is not affected by his competitors.

[2]The EU will be willing to put this goal up to 30% if the US, China and India make similar commitments.

In the electricity supply industry, an analysis without competition may not be realistic. The electricity and gas prices are actually realizing an equilibrium at each second. Thus, a new gas-fired capacity may lead to a decrease in electricity prices and an increase in gas prices, i.e., a decrease in spark spreads. The impact on electricity prices can be analyzed by comparing the marginal cost of the new capacity and the aggregate marginal cost of the current capacities. Similarly, the impact on gas prices can be analyzed by shifting the aggregate gas demand curve.

Game theory is a general approach to model decision-makings under competition. In a game theoretic framework, the first-mover advantage exists in new generation investment because decreased spark spread prices will discourage the investment of other investors. Due to the short history of deregulation, the electricity markets are usually dominated by several big suppliers. It seems reasonable to introduce oligopoly or duopoly models. With a duopoly model, the Nash equilibrium solution will give the optimal strategy for both players. It can be expected that including competition will lower the option value and the licence price. In some simple cases, scenario analysis may be a sufficient alternative approach.

7.3 Directions for Future Research

In Section 7.2, we explored the relevant issues along our research topic. These discussions laid a foundation for the possibilities of extending the research. We believe the following directions are of importance, from both the perspectives of the academia and the practitioners:

- *Richer models for the underlying variables.* To go one step ahead from Chapter 4 and Chapter 5, we can incorporate some fundamental information such as load, capacity, and temperature in the model. A hybrid model will be able to capture more information and make more accurate predictions. From Chapter 6, we can add a jump term to the spark spread model to obtain a better goodness-of-fit. Given sufficient forward data are available, we can use the forward term structure model for derivatives pricing and compare the results with those of the models based on spot prices.

- *Explicit modeling of CO_2 cost.* The CO_2 cost has been an important component in power plant valuation and investment decision-makings.

A lognormal mean reverting process may be a suitable approximation for the traded prices. In addition, the price plummet in April 2006 and the NAPs of Phase Two should be reflected by structural changes. We can also model CO_2 cost as additional real options following the spirit of [SC99].

- *Explicit modeling of competition.* Following the discussion in Section 7.2, we can analyze the changes of the aggregate supply and demand curves brought by competition and see how much they can impact the energy prices. Alternatively, we can apply the game theoretic models to a competitive market.

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James He (Chinese name "Yizhi He") was born on 5 October 1970 in Xishui County, Hubei Province in China. He attended a primary school in this county and accomplished his high school education in Huanggang Middle School in Huangzhou, Hubei Province in China. He obtained a Bachelor's degree in Economics and an MBA in Finance from Peking University. From 2002 to 2003, he stayed one year at the University of Twente, Faculty of Applied Mathematics for a visiting research in energy finance. He started his Ph.D. program in November, 2003. Before he moved in the Netherlands, he worked as a senior manager at China Netcom Corporation, Department of Equity Financing. Prior to these positions, James He had several years working experiences as a business analyst or project manager in the fields of equities, commodities, and investment banking.

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After his Ph.D. study, James He joined the APX (Amsterdam Power Exchange) Group, working as a risk analyst.

Abstract

Electricity prices are notoriously hard to model due to their exotic behaviors. The extraordinarily high volatilities, strong mean reversion, pronounced cyclical price patterns, and occasional occurrence of price spikes may demand very complicated models.

From the empirical analysis with the Dutch and German electricity market data, we find that the two-state regime switching models have a superior performance than a jump diffusion model in terms of replicating the higher moments in the historical data and forecasting electricity prices.

The real options method has been used in the valuation of energy assets and the decision-makings in operation and investment in power plants. Our empirical simulation results show that power plant values can be decreased by volumetric risk factors from both the supply side and the demand side.

Investment opportunities in power plants can be valued as American options. We value different types of investment options by using both the Hull and White trinomial tree and the Least Squares Monte Carlo method. We confirm the conclusion that the option value to invest in a peak-load power plant is higher than the option to invest in a base-load power plant when the spark spread is lower, and verse visa.